1955

Automatic gain control of transistor amplifiers

Bryan, William Littell

http://hdl.handle.net/10945/13960

AUTOMATIC GAIN CONTROL OF TRANSISTOR AMPLIFIERS

WILLIAM L. BRYAN

AUTOMATIC GAIN CONTROL

OF

TRANSISTOR AMPLIFIERS

****

William L. Bryan

AUTOMATIC GAIN CONTROL
OF
TRANSISTOR AMPLIFIERS

by

William Littell Bryan

Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
Monterey, California

1 9 5 5

This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE

IN

ENGINEERING ELECTRONICS

from the

United States Naval Postgraduate School

PREFACE

The rapid progress made in the field of semiconductors since the invention of the transistor seven years ago has widely broadened our theoretical knowledge in the field and greatly increased the potentialities of these devices. Particularly our increasing ability to manufacture reproducible transistors has brought us to the point of designing circuits for specific application to transistors, not just to illustrate the application but rather engineered to rigid specifications. This paper treats just such a design, that of obtaining automatic gain control of a transistor amplifier. There has been as yet little published about this difficult problem, and much of the work here presented is believed to be original.

The majority of the experimental work connected with this thesis was performed during the author's Industrial Experience Tour while at Lenkurt Electric Company, San Carlos, California, and he is indebted to them for their cooperation and assistance. Particular credit should go to Mr. B. R. Stack of that company for his guidance and suggestions and to Professor Abraham Sheingold of the U. S. Naval Postgraduate School for his counsel and advice on the preparation of this thesis.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (cont.)

TABLE OF SYMBOLS AND ABBREVIATIONS

AGC	Automatic gain control
Ai	Current amplification
a	Current amplification factor
a-c	Alternating-current
b	Common emitter current amplification factor
° C	Temperature in degrees Centigrade
db	Decibel
dbm	Decibels referred to 1 milliwatt in 600 ohms
d-c	Direct-current
E	D-c supply voltage
e_o	A-c output voltage
f	Frequency
$f_{\alpha o}$	Frequency at which $\lvert\alpha\rvert$ is reduced by 3 db
f_{bo}	Frequency at which $\lvert b \rvert$ is reduced by 3 db
f_{go}	Power gain cut-off frequency
G	Power gain
h_{11}	Input impedance, output short-circuited
h_{12}	Reverse voltage ratio, input open-circuited
h_{21}	Current transfer ratio, output short-circuited
h_{22}	Output admittance, input open-circuited
I	D-c current
I_b	D-c base current
I_{b2}	D-c current through second base of tetrode transistor

Ie	D-c emitter current
I_T	D-c thermistor current
i_b	A-c base current
i_e	A-c emitter current
i-f	Intermediate frequency
Li	Power loss due to mismatch
ma.	Milliampere
mv.	Millivolt
mw.	Milliwatt
n	Transformer turns ratio
n-p-n	Transistor whose base region is comprised of acceptor-type material and whose emitter and collector regions are comprised of donor-type material.
p-n-p	Transistor whose base region is comprised of donor-type material and whose emitter and collector regions are comprised of acceptor-type material.
Pb_2	D-c power dissipation of second base of a tetrode transistor
Pin	A-c input power
Po	A-c output power
Rb	External unbypassed base resistance
Re	External unbypassed emitter resistance
Rg	A-c source resistance
Rl	A-c load resistance
Rs	Shunt feedback resistance

R_T	Thermistor resistance
$R_{T_{max}}$	Maximum value of thermistor resistance
r_b	Equivalent base resistance
r_c	Equivalent collector resistance
r_e	Equivalent emitter resistance
r_i	A-c input resistance
r_o	A-c output resistance
S	Stiffness ratio of an AGC system
se	Subscript used to denote the input resistance, current amplification, and power gain when series feedback is employed
ua	Microamperes
uf	Microfarads
V	D-c voltage
Vb_2	Interbase voltage of a tetrode transistor
Vc	D-c collector voltage
Vcc	D-c collector supply voltage
Vee	D-c emitter supply voltage
v.	Volts
y	Admittance
y_l	Load admittance
y_{11}	Short-circuit input admittance
y_{22}	Short-circuit output admittance
Δy	Determinant of y matrix
$\sim$	Cycles per second

TABLE OF SYMBOLS AND ABBREVIATIONS (cont.)

Ω	Ohms
α	Short-circuit current amplification factor

CHAPTER I

INTRODUCTION

The entire field of semiconductors, which was reopened to qualitative and quantitative study during World War II after remaining dormant for many years, has advanced rapidly during the last decade. Particularly since the first announcement of the development of the transistor by J. Bardeen and W. H. Brattain in 1948, a prodigious number of papers have been published about these and other semiconductor devices. The majority of the work behind these articles has been devoted to obtaining a better and more complete understanding of the fundamental nature and operation of the devices. The results of these studies have added greatly to our fundamental knowledge and understanding of the transistor, so that today, while there yet remain many unanswered questions about solid state electronics, our ability to analyze and predict the results of experiments with transistors has increased greatly.

Much of the experimentation with transistors has also been devoted to techniques for perfecting their fabrication, primarily from the commercial point of view. Here also the results have been gratifying, and our ability to produce large quantities of transistors with predictable and uniform characteristics is advancing rapidly. The advent of low cost, mass produced, high quality

transistor production may be close at hand.

Lastly, some of this experimentation has been devoted to practical applications of the transistor. This work has been hampered in many respects by the lack of large scale production of uniform characteristic transistors. It is true that, almost as soon as the transistor was developed, engineers developed circuits and small items of equipment employing transistors in almost every imaginable application. However, much of this development has been done only to demonstrate the practicability of applying transistors to such uses, and little thorough engineering of the circuits has been done. As stated before, the big stumbling blocks to commercial development have been the high price and the irreplaceability of the transistors used in such circuits.

However, now that the production problem that has so far prevented many commercial developments of transistorized equipment is closer to being solved, attention is being placed on thorough development of practical circuits employing transistors partially or completely. In this area, experimental data on which to base design considerations is currently very sketchy or even entirely lacking; evaluation of different configurations and techniques is also quite incomplete; and reliable information concerning the variations of parameters under various operating conditions, temperatures, frequencies, etc., is almost nonexistent.

One of the requirements of many circuits and equipments employing transistors will be automatic gain control. Currently little analysis of this problem appears in the literature, and this thesis attempts to fill in the gap by presenting and discussing various methods of achieving automatic gain control of transistor amplifiers, from the viewpoint of control range, stiffness ratio, distortion, signal power levels, and control power requirements. Four possible methods of controlling the gain of a transistor are presented: use of specially constructed transistors, variation of emitter current, variation of load resistances, and variation of feedback resistances. Each of these methods has various advantages and disadvantages which makes the use of one method or of a combination of these methods depend on the specific requirements imposed on the automatic gain control system.

CHAPTER II

GENERAL CONCEPTS OF A TRANSISTOR AUTOMATIC GAIN CONTROL SYSTEM

In investigating the design of a transistorized automatic gain control system, consideration must always be given to the relative advantages of transistors and vacuum tubes. If this factor is overlooked, circuits which successfully function and meet specifications can be obtained which are quite impractical from many viewpoints as replacements for currently existing vacuum tube circuits. In particular, when considering the use of transistors in automatic gain control systems, such limitations as the low power output of transistors, their finite input power required, their frequency response cutoff at relatively low frequencies, and their sensitivity to temperature changes should all be considered in their selection. Limitations imposed by the above considerations may invalidate the advantages gained by their small size and low power consumption.

The problem of automatic gain control of transistor amplifiers is much more difficult than the corresponding problem using vacuum tubes. Automatic gain control is generally achieved in the latter through the simple expedient of constructing special tubes, variable-mu pentodes, whose output current is a function of the bias on its control

grid over a wide range of values. Thus, typically, a 30 volt change of d-c voltage on one element of the vacuum tube may produce about a 25 db variation in gain of the tube. Where extremely low distortion is required in vacuum tube automatic gain control systems, various types of bridges utilizing variable resistance elements in one arm are employed. Although they achieve low distortion and have quite high stiffness ratios, these circuits require large amounts of control power and furthermore are passive circuits and therefore involve an overall insertion loss.

The approaches required for transistor circuits are not so direct. Inspection of collector characteristic curves for both the common emitter and common base configurations reveals that the constant emitter curves are closely parallel and evenly spaced, a desirable condition for low distortion but a disadvantage from the automatic gain control viewpoint, since it seems to indicate that gain cannot be varied by changing the operating point.

As far as construction is concerned, there is far less latitude available to the transistor designer and a variable control element is quite difficult to introduce into the device. It is true that transistors have been constructed which permit a certain measure of control of gain by varying the current into one element, but they are as yet difficult to manufacture and more difficlut to predict as far as response is concerned.

A further problem of transistor AGC systems is that, while vacuum tube systems require only a control voltage and negligible control power, transistors require a definite input power, part of which may be furnished by the control system, thereby sapping some of the already limited power output of the amplifier.

Before proceeding into the investigation of the application of automatic gain control to transistor circuits, it would appear best to determine exactly what characteristics it would be desirable to obtain. Ideally an automatically-gain-regulated amplifier should produce a constant level output despite large changes of input level. Practically we must accept some change in output level, since gain regulating systems are Type 0 servo systems requiring that the gain controlling level vary as a function of the amplifier output level. It remains then to ascertain just how much variation of output level is acceptable in any application, and this then determines one specification of the AGC system. For this purpose the stiffness ratio of an AGC system is defined as the ratio of the change in input level in db to the change in output level in db that it produces. Another specification for an AGC system might be that for a delay feature so that receiver sensitivity would not be reduced for weak carrier signals. Further specifications would delineate the allowable distortion, the desired range of frequency response, the

input and output power level, and the maximum power available from the amplifier output for use in the control circuit.

In radio communication systems a typical specification for an AGC system might be a 6 db change of output level for a 60 db change of input level, a stiffness ratio of 10. Distortion levels of -26 db or greater might be tolerated; the power levels present will vary widely depending on the application of the circuit, as will also the desired frequency band. In many applications the AGC system must control only a single frequency, as the i-f frequency in a receiver. A delay feature may often be required.

The specification for a carrier telephone system, while covering a smaller range of input power variations, might be more severe in other aspects. A typical requirement might be a 2 db change in output level for a 30 db change of input level, a stiffness ratio of 15, relaxing for larger input variations to a 4 db output level change for a 50 db change of input level, a stiffness ratio of 12.5. The maximum allowable distortion would normally be in the nature of -65 db, but with the use of compandors this requirement could be eased to about -43 db. Receivers in this application must have a flat frequency response over a wide band of relatively low frequencies; their power output (generally applied to a demodulator) must be of the nature of -20 dbm to -30 dbm, with input power levels

ranging up to 0 dbm, a much higher level than would be encountered with a radio communications receiver.

The maximum power available from the output for use in the control system without excessively draining the available power output of the amplifier would be in the nature of +3 dbm to +6 dbm for both applications.

The above values give an order of magnitude for specifications of an automatic gain control system and serve as a goal to be reached (and exceeded if possible) in designing a transistorized automatic gain control system which could practically replace a vacuum tube circuit.

CHAPTER III

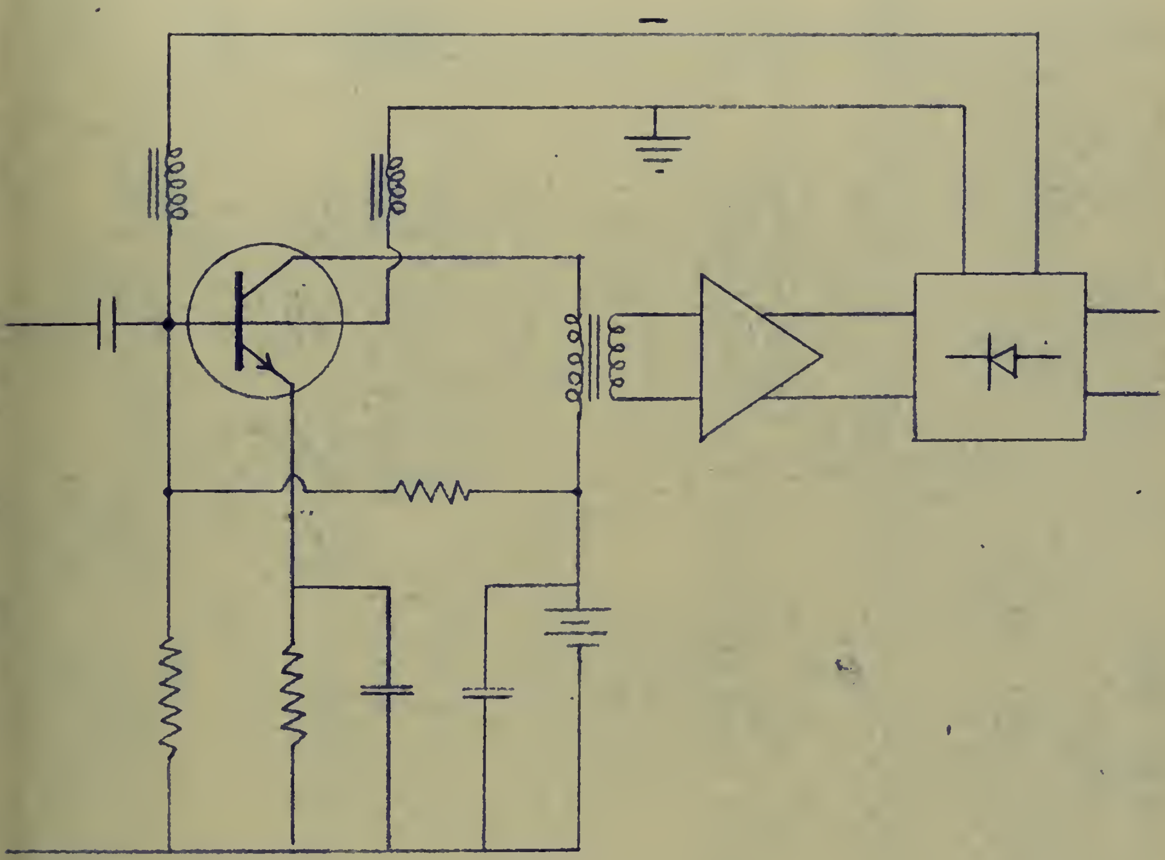
AUTOMATIC GAIN CONTROL EMPLOYING TETRODE TRANSISTORS

As mentioned previously, the tetrode transistor is a specially constructed transistor having a controllable gain characteristic. This special transistor has immediate appeal for use in an automatic gain control system because of its analogy to the variable-mu pentodes so commonly used in present-day receiving systems. The device was initially constructed with a view to obtaining higher alpha cutoff frequencies through the medium of reducing the base resistance of the transistor by forcing the base current to flow through a restricted area of the base, and also by reducing the collector capacitance (even though thinner base regions are used), again because of the decreased area of base current flow. [14] This is accomplished in this device through the use of a second base connection with an interbase bias of a few volts being applied. Apart from considerations of high frequency response, the effect of the interbase bias is to create a transverse electric field within the base region which causes the minority carriers to drift through the base region towards one or the other of the bases. The deflection of such carriers from the collector junction and their subsequent recombination within the base effectively

decreases the proportion of the injected emitter current which reaches the collector junction and thus reduces alpha. Since the gain of a common base circuit is approximately a direct function of alpha $\left[G = \frac{\alpha^2 R_\ell}{r_e + r_b(1-\alpha)} \right]$, interbase bias in either direction acts as an effective controller of the gain of the amplifier.

The degree of control achievable by this method would of course depend a great deal upon the actual construction of the tetrode transistor, being a function of the method of fabrication, the degree of doping of the base region, the geometry of the base region and the location of the base contacts. It will also vary to some extent with the emitter current, having more control with low values of emitter current. By the proper selection of emitter current, the desired degree of control could be obtained through a system somewhat as shown in Figure 1.

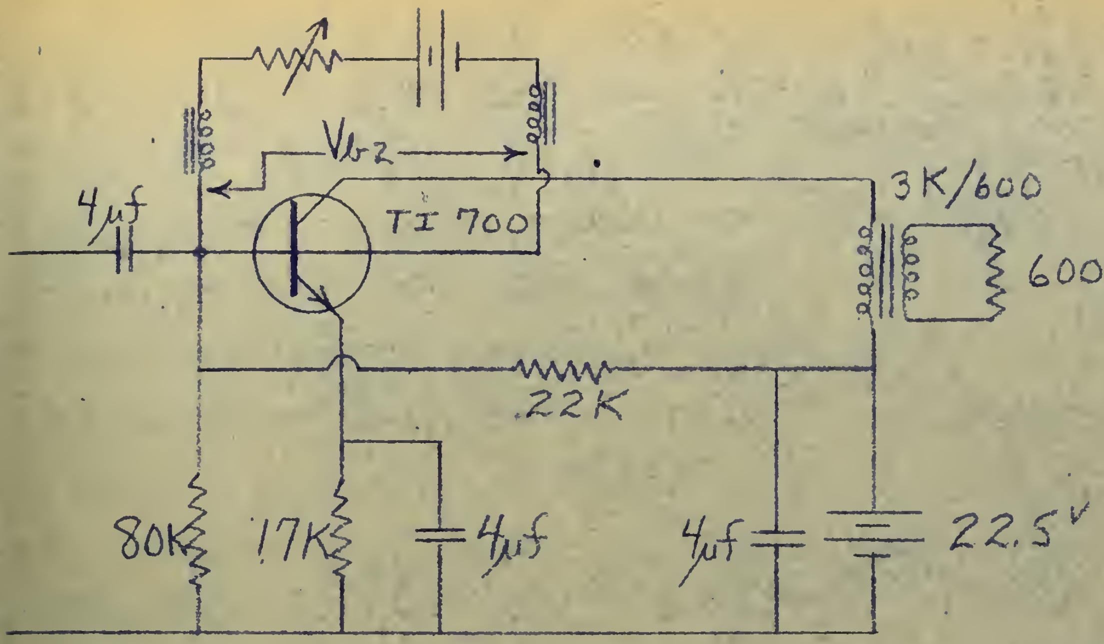
Due to the thinness of the base region, tetrodes are difficult to fabricate, and even more difficult is the task of attaching two base connections to this thin area. As a result of these problems, tetrode transistors currently being manufactured have widely varying characteristics. An experiment run on a small number of tetrode transistors produced by one manufacturer (see Figure 2) to determine the maximum possible change of gain obtainable for one setting of emitter current produced gain variations of between 9 and 16 db, not an extremely wide range for



AGC System

Employing a Tetrode Transistor

Figure 1



Vc = 5.0 v.
Ie = 1.0 ma.

f = 1 KC
Output Level = 0 dbm
for Ib_2 = 0 ma.

Unit No.	Insertion Gain for Ib_2 = 0 (db)	Change of Gain (db)	Ib_2 (ma.)	Vb_2 (v.)	Pb_2 (mw.)	Distortion (db)
1	32.0	9	2.1	3.2	6.72	- 24
2	32.7	16	2.0	2.7	5.4	- 27
3	33.3	16	2.4	3.0	7.2	- 23
4	34.3	16	2.8	1.5	4.2	- 24
5	34.8	10	2.2	1.2	2.6	- 29
6	32.5	16	1.5	2.35	3.5	- 25

Test Data on TI 700 Tetrode Transistors

to determine the maximum change of gain available, the control power required, and the output distortion

Figure 2

such a device. As can be seen from the experimental data, there is considerable variation between units as to insertion gain, range of control available, control power required, and intergrid bias. The manufacturer's specifications for these devices claims upwards of 20 db attenuation by introducing less than 100 microamperes into the second base lead. That these half dozen units all failed to meet the specifications by a large amount speaks for the difficulty of manufacturing these tetrodes with high quality and interchangeability.

One of the biggest advantages of using tetrode transistors, besides the simplicity of the control system, is the fact that they operate at a constant bias point and thus the equivalent circuit parameters remain constant. Further the bias point may be chosen at values of Ie and Vc such that for small signals the distortion produced is small. Though the distortion is relatively large for the test data given, this is due to the high power levels being used and would be much less for lower power level inputs without requiring that these input levels be excessively low. The fact that the test was run at about the maximum output of the tetrode points out the limitation that these devices have in power handling capability, due to the thinner base region and the restricted area of the base through which the current flow takes place. A further disadvantage of the device, in addition to its high cost

and lack of replaceability, is the appreciable amount of control power required, this of course being subtracted from the output power. Currently these disadvantages disqualify tetrodes practically; however, with improved production techniques, this special transistor could become a practical and effective device to employ as a gain controlled amplifier.

CHAPTER IV

AUTOMATIC GAIN CONTROL

BY VARIATIONS OF EMITTER CURRENT

Another possible method of controlling the gain of a transistor amplifier is through varying the value of the d-c emitter current supplied to it. The change of gain obtained is small for emitter current variations in the region above about one milliampere but becomes much greater for variations of Ie in the region below one milliampere. That this method would produce a variation in the gain of the amplifier is not immediately obvious from glancing at the formulae given for computing the gain of a transistor amplifier.

As can be derived easily, these gains for the various configurations are: [8, pp. 61-63]

Common base: $$\frac{a^2 R_l}{r_e + r_b(1-a)} \qquad (4\text{-}1)$$

Common emitter: $$\frac{a^2 R_l}{(1-a)\left[r_e + r_b(1-a)\right]} \qquad (4\text{-}2)$$

Common collector: $$b+1 = \frac{1}{1-a} \qquad (4\text{-}3)$$

When: $r_c >> r_b$

$r_c(1-a) >> R_l >> r_e$

Although these formulae are independent of the emitter current and collector voltage directly, it is commonly known that the values of the equivalent circuit parameters are all functions of the biasing conditions and therefore will vary as the emitter current is varied. In Appendix I are a few typical curves showing the variation of the equivalent circuit parameters with emitter current for a silicon transistor and two germanium transistors. The data is presented with no pretense of representing average characteristics, since it represents only three individual transistors: rather it indicates a trend in the variation of the parameters of the transistors tested. The various biasing points selected for the different curves are typical of the values of Ie and Vc obtained in experimental work upon which this thesis is based. For more accurate and detailed data on the variation of transistor parameters the manufacturer's specifications and statistical curves and various textbooks should be consulted. [8, 9]

From the curves it can be seen that the short circuit amplification factor (alpha) decreases slightly as the emitter current is decreased. From other data published, alpha is known to decrease sharply at emitter currents below about 10 ua and again to decrease somewhat more gradually for very large values of emitter current. The reasoning behind this variation is discussed quite

adequately by Webster [15], and is not germane to this paper. Glancing again at the gain equation given above, one can see that this variation in alpha will definitely cause a decrease in gain (reference is made in this thesis only to junction transistors where the approximation that alpha equals "a" is quite legitimate. [8, 11]) This will be particularly true for the common emitter and common collector configurations which contain the factor 1/1-a which varies much faster than "a" itself does. However, the variation of alpha observed is too small to produce any large variations of gain: for the germanium transistor varying alpha between 0.90 and 0.95 would produce only a 0.5 db change of gain due to the change of "a^2" and only 3 db due to the change of 1/1-a, and for the silicon transistor varying alpha between 0.625 and 0.93 would produce only a 3.4 db change due to the change of "a^2" and 7.3 db due to the change of 1/1-a. Were alpha the only parameter that changed with decreasing emitter current, this method of gain control would be impractical, but fortunately other factors must be taken into account.

The rapid increase of emitter resistance with decreasing emitter current as predicted by theory [8, 11], is quite obvious from the curves. More difficult to ascertain is the trend of the base resistance as the emitter current is varied. This is due primarily to the difficulty in obtaining an accurate value of h_{12}, the backward voltage

ratio with the input open circuited, on the particular test set employed in obtaining these measurements. Typical curves given in various texts indicate that the base resistance increases with decreasing emitter current, which tendency is borne out by the data for two of the three transistors measured here. In any event, the ratio of the maximum to the minimum value of r_b experienced is no greater than 2.0 and we may consider the effect of the variation of the base resistance one of producing only a slight variation of power gain of the amplifier. On the other hand, the ratio of the maximum to the minimum values of emitter resistance obtainable is about 20 for the germanium transistors and about 30 for the silicon one. This variation will have a considerable effect on the gain of the amplifier, capable of producing in the neighborhood of 10 db or more change of gain.

A further effect must also be noted. The variation of parameters due to a shift in the biasing point of the transistor not only produces a change in the gain of the device but also changes the input and output impedance of it. Let us consider the input impedance, which may be calculated from the formulae below:

Common base: $$r_e + r_b(1-a) \qquad (4\text{-}4)$$

Common emitter: $$r_b + \frac{r_e}{1-a} \qquad (4\text{-}5)$$

Common collector: $$(b+1) r_e = \frac{r_e}{1-a} \qquad (4\text{-}6)$$

When: $r_c \gg r_b$

$r_c(1-a) \gg R_l \gg r_e$

The power loss due to a mismatch between a constant voltage source V with internal impedance r_g and a circuit having an input impedance r_i is:

$$L_i = \frac{\left(\frac{V}{r_g + r_i}\right)^2 r_i}{\left(\frac{V}{2 r_g}\right)^2 r_g} = \frac{4 r_g r_i}{(r_g + r_i)^2} \qquad (4\text{-}7)$$

(This is also the power loss due to a mismatch between a constant current source I shunted by an internal impedance r_g and a circuit having an input impedance r_i:

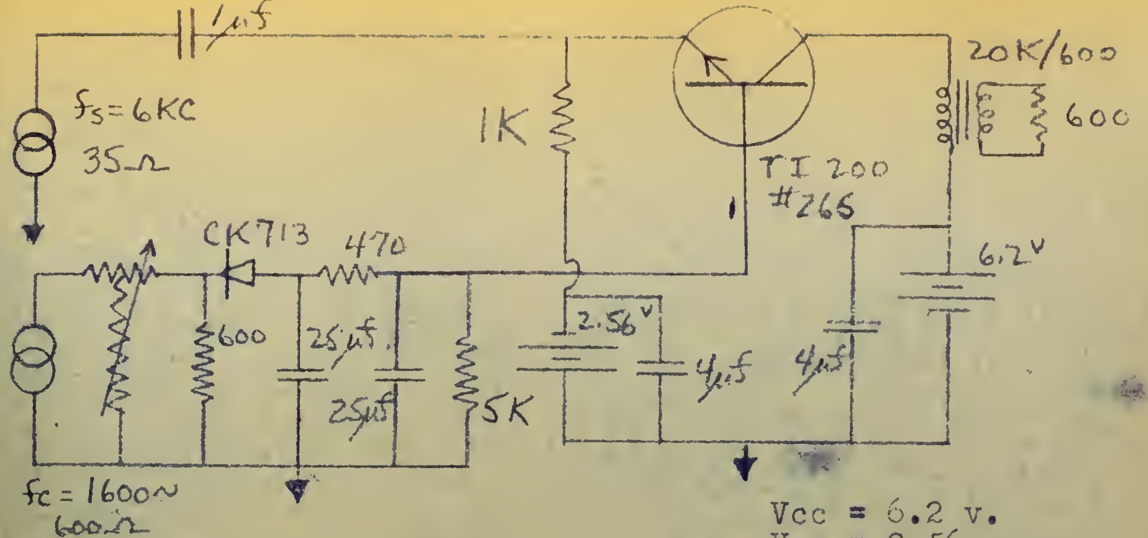
$$L_i = \frac{\left(\frac{I}{\frac{r_g + r_i}{r_g r_i}}\right)^2 \frac{1}{r_i}}{\left(\frac{I}{\frac{2 r_g}{r_g^2}}\right)^2 \frac{1}{r_g}} = \frac{4 r_g r_i}{(r_g + r_i)^2}$$)

Since from previous considerations we know that "a", r_b, and r_e all vary with decreasing emitter current, we can easily see that r_i will also vary with it and so also will the power loss due to improper match at the input. If $r_i \gg r_g$, this power loss variation as Ie is decreased is such as to add to the change in gain of the transistor.

It may thus be seen that there is a decrease in insertion gain as the emitter current is decreased due to

an actual decrease of transistor power gain because of the variation of the equivalent circuit parameters and further due to the variation of the degree of mismatch between the source and the amplifier. An experiment was performed to ascertain how closely the above theory compared to the actual gain variations obtainable. In the circuit shown in Figure 3, the emitter current was varied between 30 ua and 300 ua through varying the control attenuator; the resulting changes of gain were recorded as shown in the data. Then for the values of Ie and Vc used in this test, the hybrid parameters of the transistor used were measured and the equivalent circuit parameters computed (see Test G, Appendix I, for the data for this measurement and computation). Then using these values of the equivalent circuit parameters, the theoretical insertion gain was computed for the various emitter currents used. Comparison between the theoretical and actual values of insertion gain shown in Figure 3 does not show any great degree of correlation between them, but when the theoretical and actual insertion gain changes as a function of the emitter current are plotted (Figure 4), the two curves are seen to be almost coincident, indicating that the above analysis truly predicts the observed gain variations.

There are two limitations that become obvious in studying this method of gain control. The first of these



Vcc = 6.2 v.
Vee = 2.56 v.
Pin = - 36.0 dbm

Ie (ua.)	Vc (v.)	e_o (mv.)	Po (dbm)	Distortion (%)	Insertion Gain (db)	Change of Insertion Gain (db)
300	5.50	60.0	- 22.2	3.33	13.8	--
175	5.75	42.2	- 25.3	4.41	10.7	3.1
125	5.90	32.8	- 27.5	4.97	8.5	2.2
100	5.95	26.9	- 29.2	----	6.8	1.7
50	6.05	20.7	- 31.5	6.77	+ 3.5	2.3
30	6.15	9.8	- 38.0	----	- 2.0	6.5

Total: 15.8 db

Theoretical Calculations

(for values of r_e, r_b, r_c, and alpha obtained for the values of Ie and Vc measured above and used in these computations below, see Test C, Appendix I)

Ie (ua.)	Amplifier Gain (db)	Input Resistance (ohms)	Input Mismatch Loss (db)	Insertion Gain (db)	Change of Insertion Gain (db)
300	16.36	100.0	1.15	15.21	--
175	13.38	193.9	2.71	10.67	4.54
125	12.84	217	3.20	9.64	1.03
100	11.85	271	3.92	7.93	1.71
50	9.06	502	6.15	2.91	5.02
30	8.46	570	6.62	1.84	1.07

Total: 13.37 db

Comparison of Theoretical versus Actual Insertion Gain as a function of Emitter Current

Figure 3

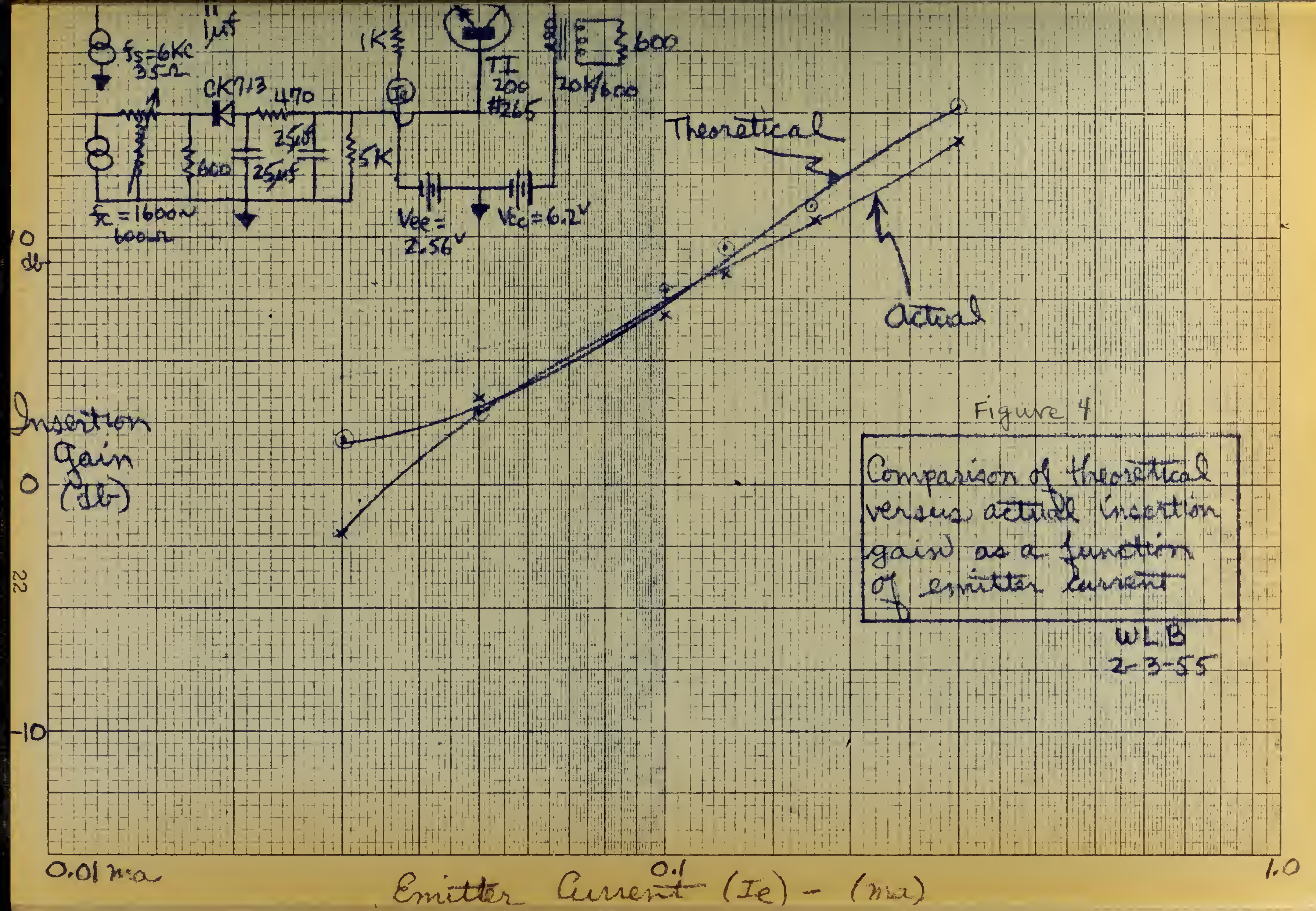


Figure 4

Comparison of theoretical versus actual insertion gain as a function of emitter current

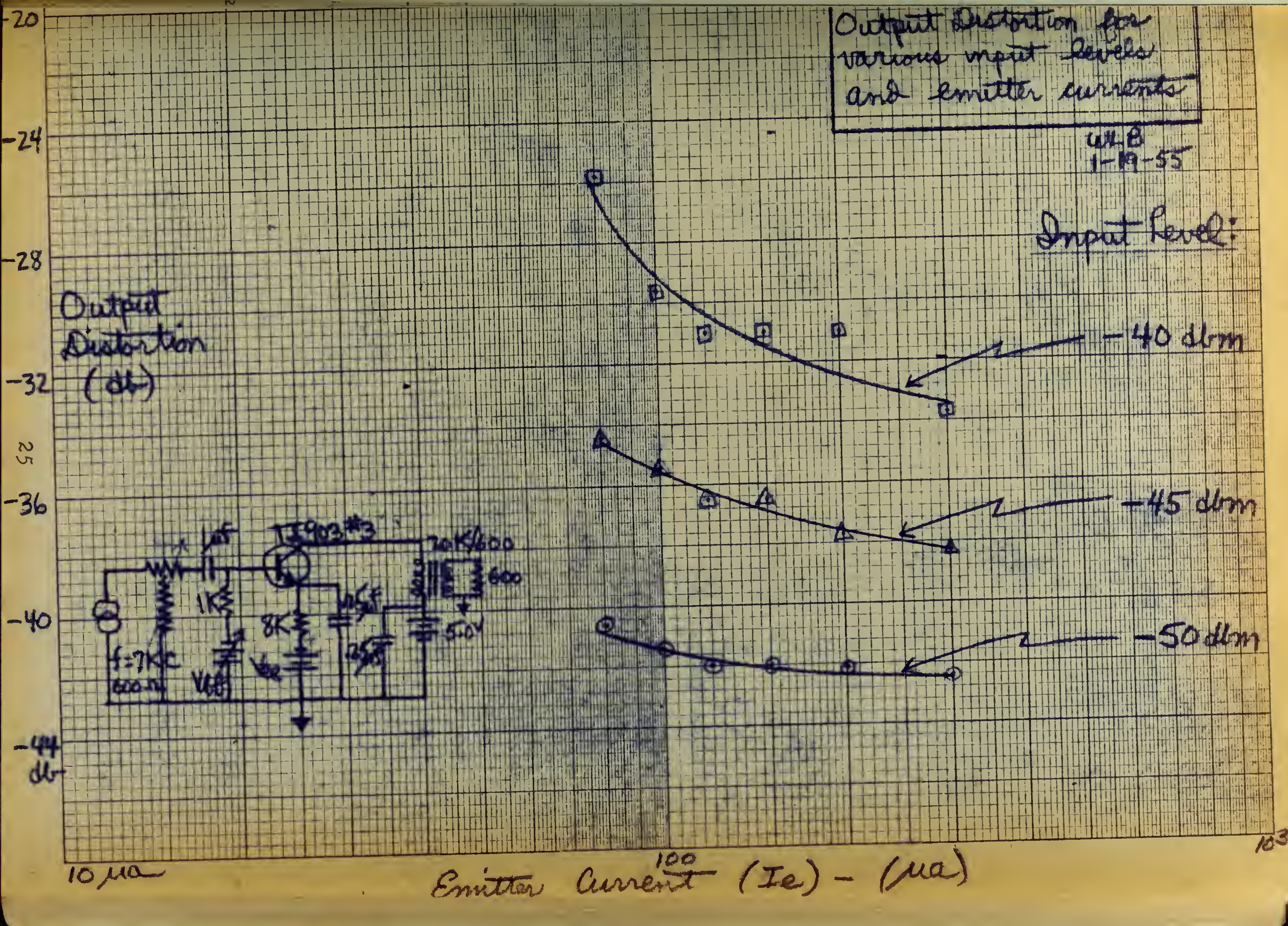
WLB
2-3-55

is the well-known variation of several of the equivalent circuit parameters with temperature. Most critical of these would be the exponential variation of the collector cutoff current with temperature. Without elaborate stabilization systems the increase of this current can change the operating point of the transistor, particularly when the transistor is being operated at low emitter currents, and thus all control will be lost. Alpha, base resistance, emitter resistance, and collector resistance also all vary with temperature, although below about 50° C these variations can be neglected for this application. On the application of heat to several different circuits being tested under conditions where the emitter current was of the order of 10 ua. to 50 ua., the effect on germanium transistors was a rapid increase of emitter current and complete loss of control. With silicon transistors, however, there was only a very small increase of emitter current observed and control was maintained. Thus, under conditions requiring any large variation of ambient temperature, silicon transistors would be required for use in this type of automatic gain control system.

The other limitation of this system is the maximum input signal which can be used without the creation of excessive distortion in the output. This distortion is caused primarily by the variation of the insertion gain as the emitter current is varied. This is, of course,

the desired effect here and thus this distortion is inherent in this method of automatic gain control. For very small signals the distortion created by variations about any bias point is not excessive, but as the amplitude of the signal increases the distortion increases quite rapidly, particularly at low values of emitter current, as shown in Figure 5. In this figure, the distortion for various input levels and for various values of emitter current were obtained. For a communications receiver, a distortion as high as -26 db might be allowed for one stage, although an attempt would be made to hold it down below this level. Thus the distortion of this system is not a serious limitation for a communications receiver, since the distortion at these levels is not excessive; and further, the levels used are slightly higher than those normally encountered in the gain controlled section of a receiver. For a carrier telephone system, where the input levels to the gain controlled section of the receiver are in the nature of -50 dbm to 0 dbm, this method of controlling gain will not prove practical if distortion specifications of at least -43 db are to be met.

Some mention should also be made of the methods for controlling the emitter current of a transistor. The best method would appear to be that of controlling the potential of the base with the emitter potential negative and the collector potential positive for a n-p-n transistor and



Output Distortion for various input levels and emitter currents
W.H.B
1-19-55
Input Level:
-40 dbm
-45 dbm
-50 dbm
Output Distortion (db)
-20
-24
-28
-32
-36
-40
-44 db
10 μa
100
10^3
Emitter Current (Ie) - (μa)
1μf
1K
8K
f=7KC
600 Ω
20K/600
600
5.0V

vice versa for a p-n-p transistor. The control voltage on the base and the emitter voltage must have the same polarity. The advantage of this method of controlling the emitter current lies in the fact that the low values of current flowing in the base lead (as compared to the larger current flows in the emitter and collector leads) makes this a high impedance point and therefore the power requirements of the automatic gain control circuitry are minimized. Variations of the base potential also have a differential effect of varying Ie and Vc in opposite directions, but this appears to have little effect on the variations of gain.

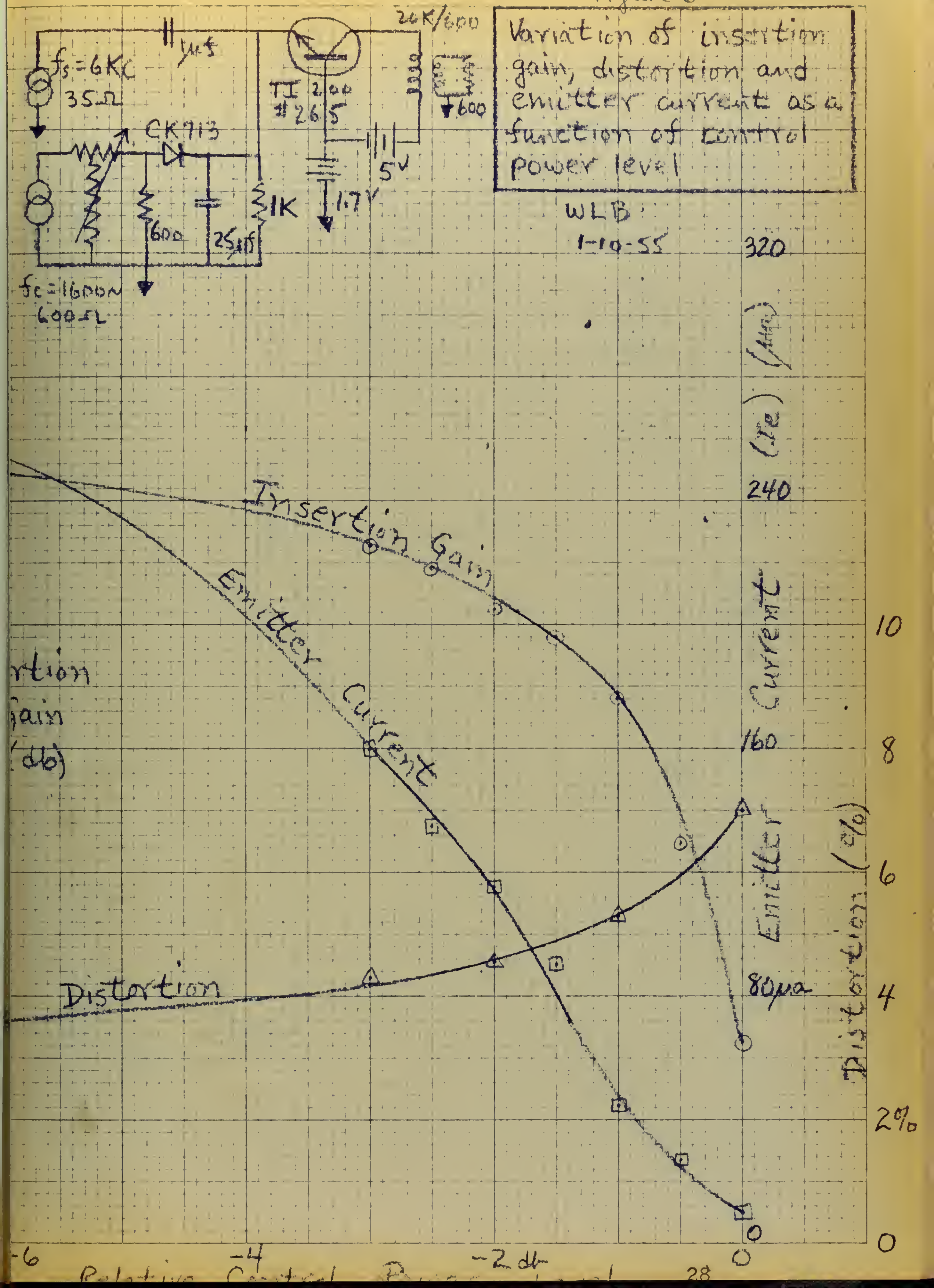
The use of the common emitter versus the common base configuration seems to offer little noticeable difference as to the degree of control possible, and this thought is borne out by experimental work. The distortion obtained with the common emitter circuit, however, would be expected to be slightly higher than that obtained when the common base circuit is employed; similarly the frequency response of the common emitter circuit is somewhat poorer than that of the common base circuit due to the much lower power gain cutoff frequency of the former configuration. The variations of gain obtained when substituting various transistors with different alphas in the common emitter circuit, coupled with these other factors, all recommend the use of the common base circuit.

Various experimental data was obtained to support the above ideas and to ascertain what actual range of control and what amount of distortion would be produced. Typical results obtained using a common base configuration and injecting a control voltage in the emitter lead or the base lead are given in Figures 6 and 7 respectively. It is seen that the range of control obtainable for a 2 db change of control voltage is slightly greater (by about 2 db) for the latter method, although the distortion produced by this method is unfortunately also higher by about 3 db. But the greatest advantage of this latter method is that the power requirements were slightly lower.

A typical test result for a common emitter circuit employing a controllable base voltage is also shown (Figure 8) Although the range of control of this particular test is lower than that obtainable using the common base configuration, comparable gains could be obtained. Of interest in this test is the fact that the distortion obtained was noticeably lower than that observed in the above tests, due to the slightly lower input level used.

Shown in Figure 9 is the result of tests to ascertain the range of control available by this method of automatic gain control without exceeding distortion limits of -40 db and -43 db. As can be seen from the data, the ranges of control available are 10.3 db and 7.0 db respectively, and the maximum input levels were -38.4 dbm and -50.2 dbm.

Figure 6


Variation of insertion gain, distortion and emitter current as a function of control power level
WLB
1-10-55
fs=6KC
35Ω
1µf
TI 200 #265
24K/600
600
CK713
1K
600
25µf
5V
1.7V
fc=1600~
600Ω
Insertion Gain
Emitter Current
Distortion
Insertion gain (db)
Emitter Current (µa)
Distortion (%)
320
240
160
80µa
10
8
6
4
2%
0
-6
-4
-2 db
0

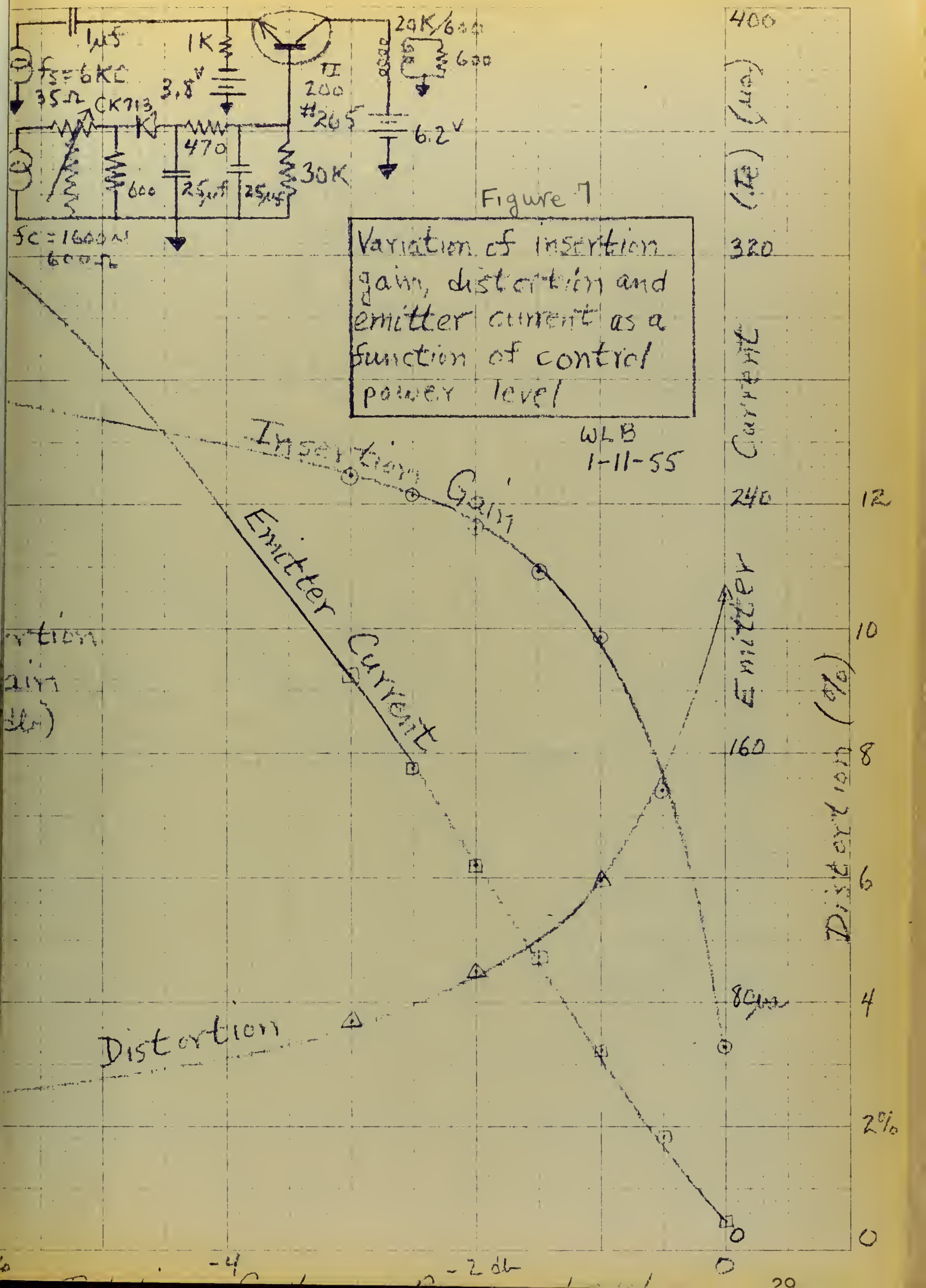
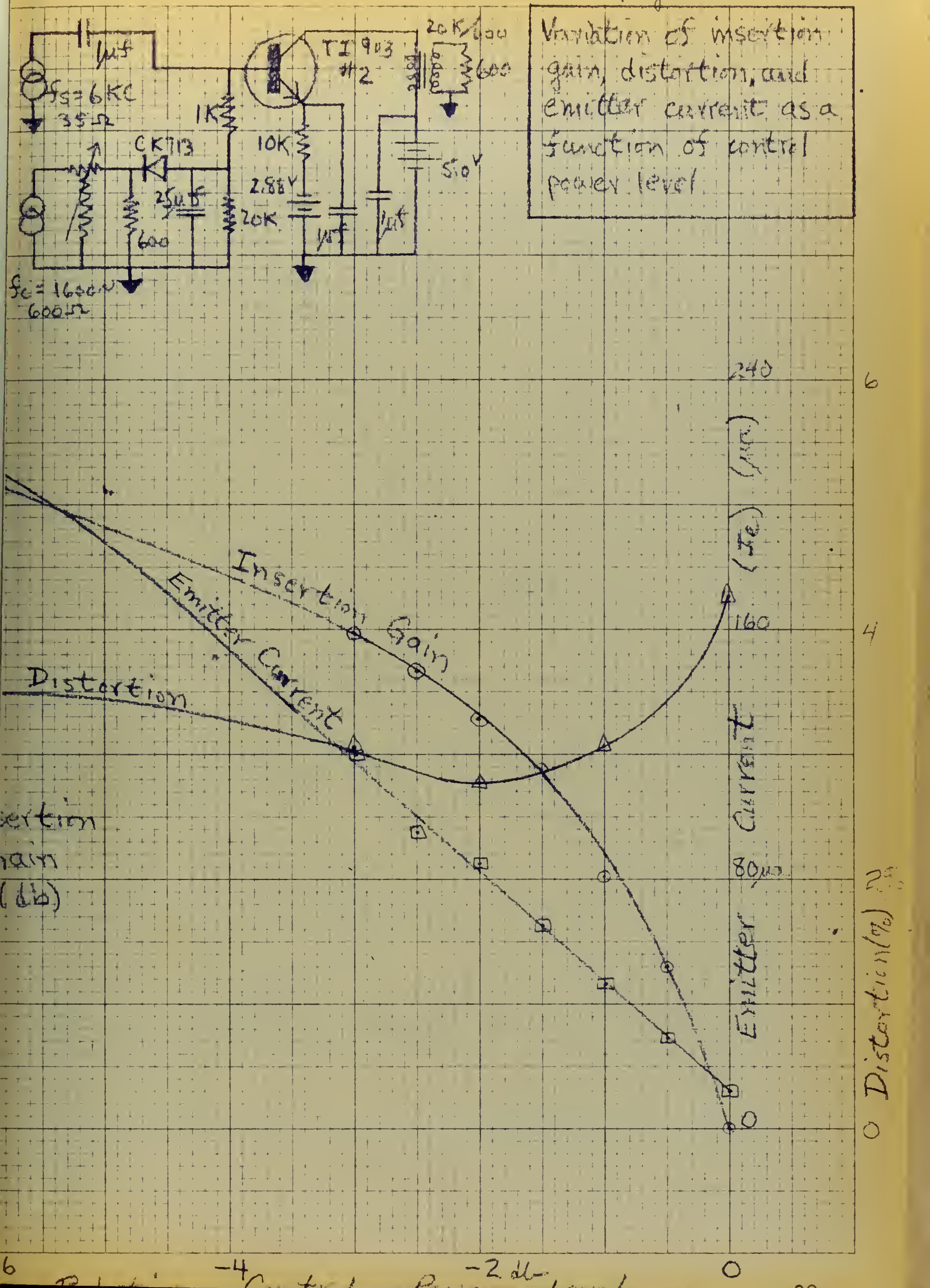
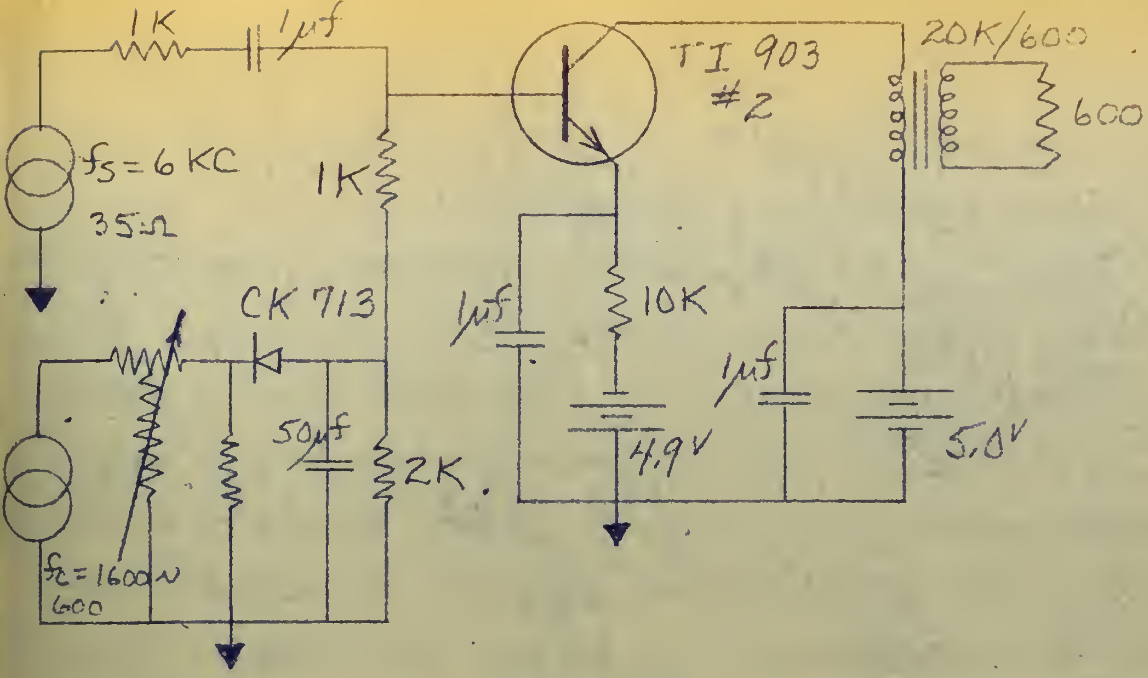


Figure 7

Variation of insertion gain, distortion and emitter current as a function of control power level

Figure 8


Variation of insertion gain, distortion, and emitter current as a function of control power level
1µf
$f_s = 6KC$
35Ω
1K
CK713
25µf
600
20K
$f_c = 1600$
600Ω
TI 903 #2
10K
2.88V
1µf
20K/600
600
5.0V
Insertion Gain
Emitter Current
Distortion
Insertion Gain (db)
240
160
80µa
0
Emitter Current (I_e) (µa)
6
4
2
0
Distortion (%)
-6
-4
-2 db
0
Relative Control Power Level



Pin = - 38.4 dbm

Relative Control Power Level (db)	Emitter Current (μa.)	Signal Output Level (dbm)	Distortion (%)
- 20	335	- 32.7	0.918
- 16	235	- 32.7	0.918
- 10	230	- 32.7	0.918
- 6	180	- 34.7	1.058
0	75	- 43.0	0.819

A change of insertion gain of 10.3 db was obtained for a ten db change of control power level with a maximum distortion of - 39.5 db.

Pin = - 50.2 dbm

- 13	3,000	- 24.3	0.524
- 10	1,350	- 24.3	0.524
- 6	[illegible]50	- 26.5	0.583
- 3	500	- 30.2	0.[illegible]7
0	300	- 31.3	0.763

A change of insertion gain of 7.0 db was obtained for a thirteen db change of control power level with a maximum distortion of - 42.0 db.

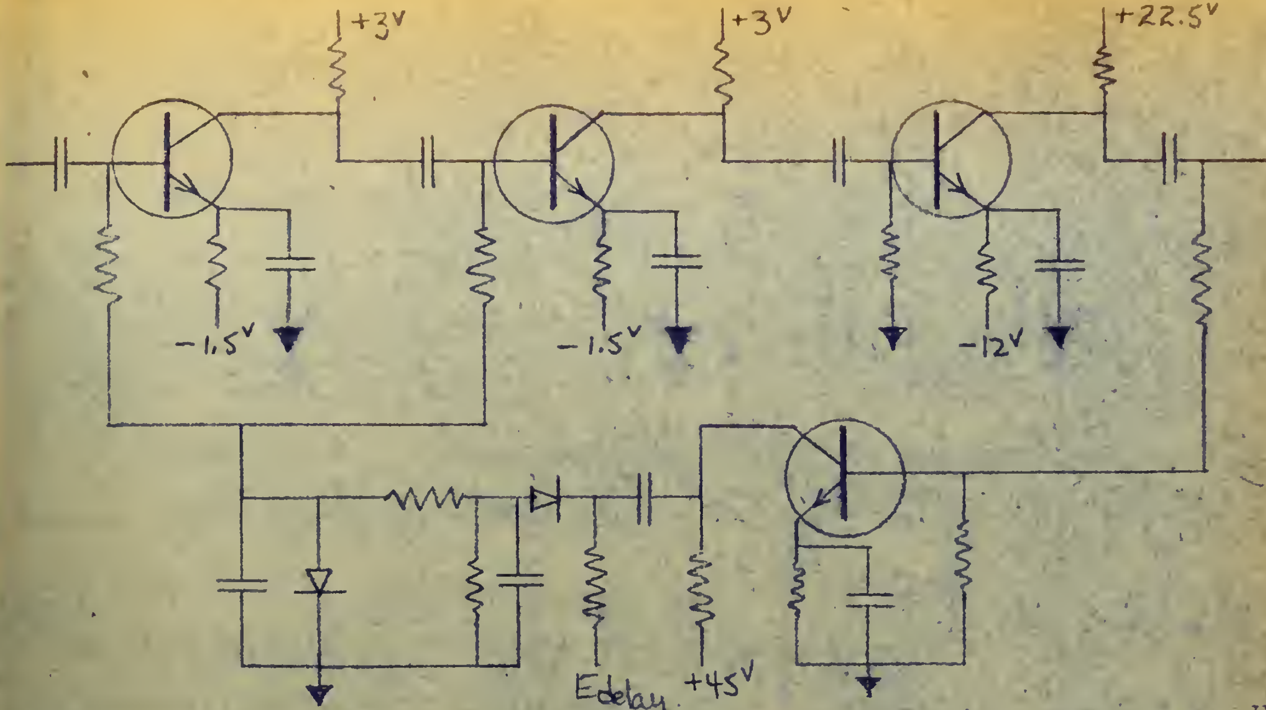
Variation of insertion gain obtainable with a maximum distortion of 1% and 0.8 %

Figure 9

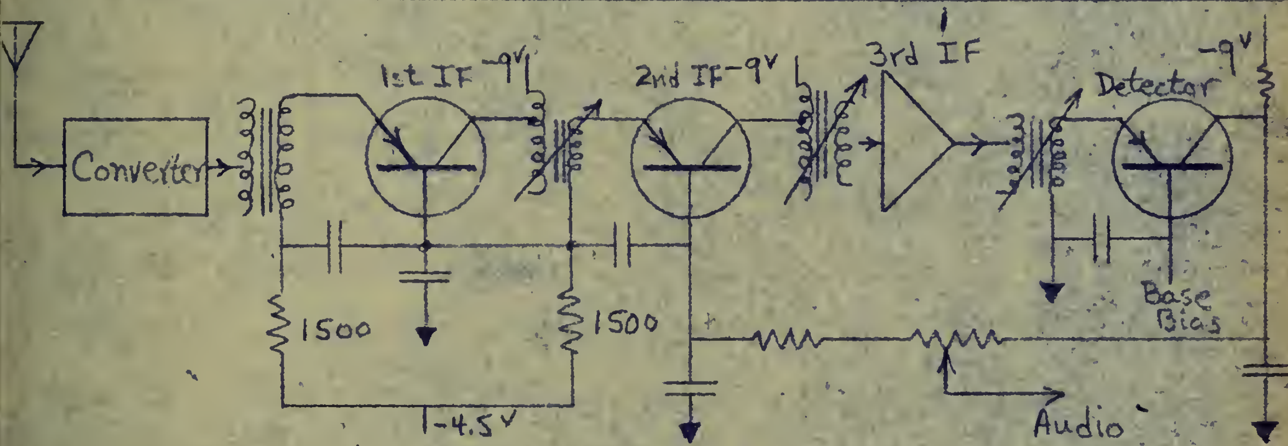
These figures indicate the difficulties involved in attempting to obtain low distortion in the gain control system and yet maintain a reasonable range of control.

The test results in general show that gain variations of the order of 17-20 db can be obtained from either the common base or common emitter circuits with only a 2 db change in control voltage. However, this involves variations of the emitter current down to values of the order of 10 ua. and the output distortion ranging up to -23 db to -30 db. For lower distortion requirements the minimum usable emitter current is increased and the range of control available is correspondingly decreased. Due to the effect of temperature changes, silicon transistors are preferred for use in these circuits and are a necessity when the emitter current is decreased to a few microamperes.

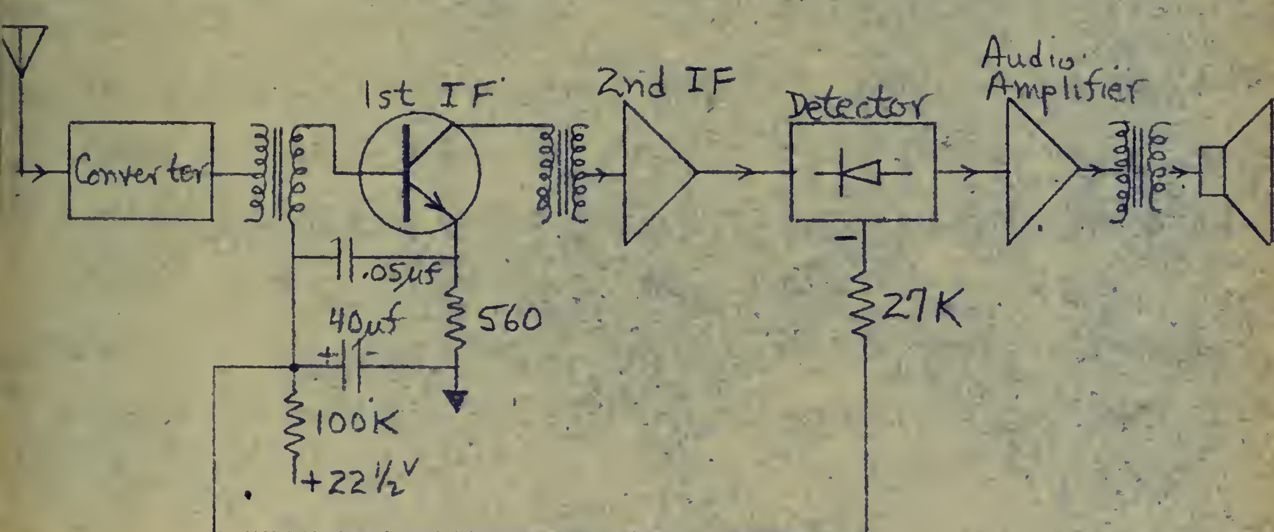
A system for automatic gain control of a multistage audio amplifier basically employing this method of controlling the emitter current of the amplifier is described in the Johns Hopkins notes on transistors. [7] This system employs three common emitter amplifiers with an automatic gain control amplifier (with variable delay available) to supply a base voltage controlling the gain of the first two stages. (Figure 10a) Neither the values of emitter current employed nor the distortion produced are quoted, although the latter is stated to be negligible. Experimental work on similar stages produced distortion ranging



Automatic Gain Controlled Audio Amplifier designed by Johns Hopkins Univ.



Laboratory Model of a Commercial Broadcast Receiver -- RCA



c. Regency TR-1 Commercial Broadcast Receiver

Figure 10

from about -15 db to -40 db depending on input level, which would be considered negligible for many applications. From the curves given for the circuit, the stiffness ratio is seen to be about 5 with a 6 db change of output for a 30 db change of input level.

A slightly different variation of this method is found in an RCA laboratory model of a broadcast receiver. [1] In this receiver the AGC voltage is supplied from the detector and applied to the base of the second i-f stage, a common base amplifier. (Figure 10b) This voltage supplies the emitter current for the stage and thus as the AGC voltage decreases so also does the emitter current and the stage gain. Further, the voltage drop due to the emitter current across a resistor in the emitter lead is used as a control voltage on the base lead of the first i-f stage. Thus the decrease of emitter current in the second i-f stage due to the decrease of AGC voltage causes a decrease of emitter current in the first i-f stage. An AGC voltage which decreases with an increase in signal level is achieved through the decrease of Vc of the detector due to the increased voltage drop across the resistor between the detector and Vcc as the input level decreases. This tandem arrangement of gain control is used here to place a minimum d-c load on the detector and to decrease the gain of the first i-f stage faster than that of the second i-f stage in order to keep distortion at a minimum. Here again the

distortion produced is not stated; the value of emitter current employed in the i-f amplifier is given as 0.5 ma. and in the detector as 0.2 ma. The automatic gain control characteristic obtained showed a 6 db change of output level with a 40 db change of input level, a stiffness ratio of 6.7. The characteristic was actually better than that of a similar battery-operated vacuum tube receiver.

A further example of the use of this method is found in the Regency TR-1, a recently developed commercial broadcast receiver. [13] Here the detector d-c voltage is applied to the base of the first i-f stage where it opposes the positive bias in this lead and thus reduces the emitter current of this stage as the output level of the i-f amplifier tends to increase. (Figure 10c)

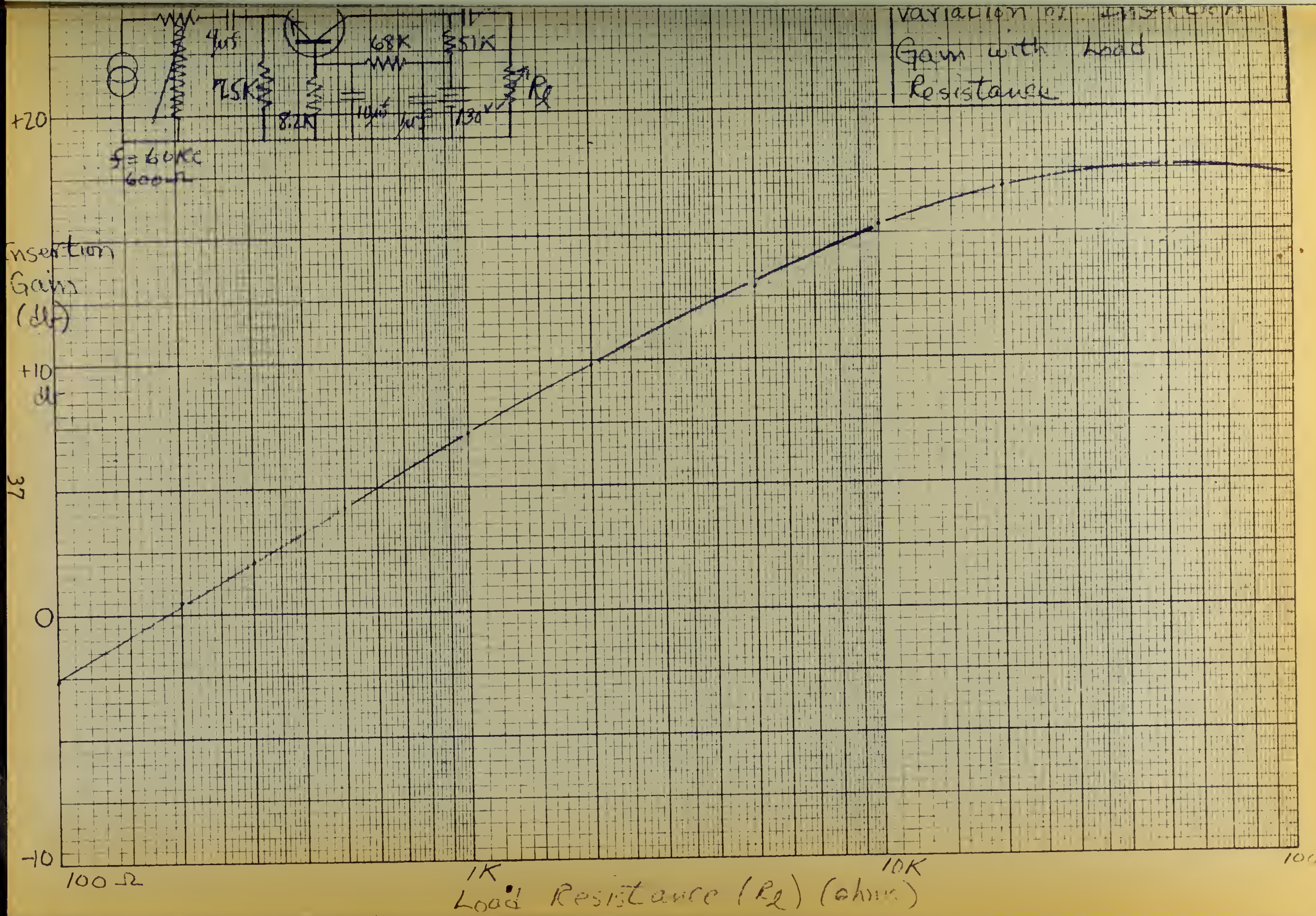
These few applications of this method of controlling gain demonstrate the practicability of it for use in broadcast receivers and suggest that it could be applied to other communications receivers requiring greater stiffness ratios and wider range of control through the use of more elaborate methods of varying the emitter current. In these applications the distortion inherent in the system is not great enough to be disqualifying, but if a low distortion control (below -40 db) is desired, some other method must be employed.

CHAPTER V

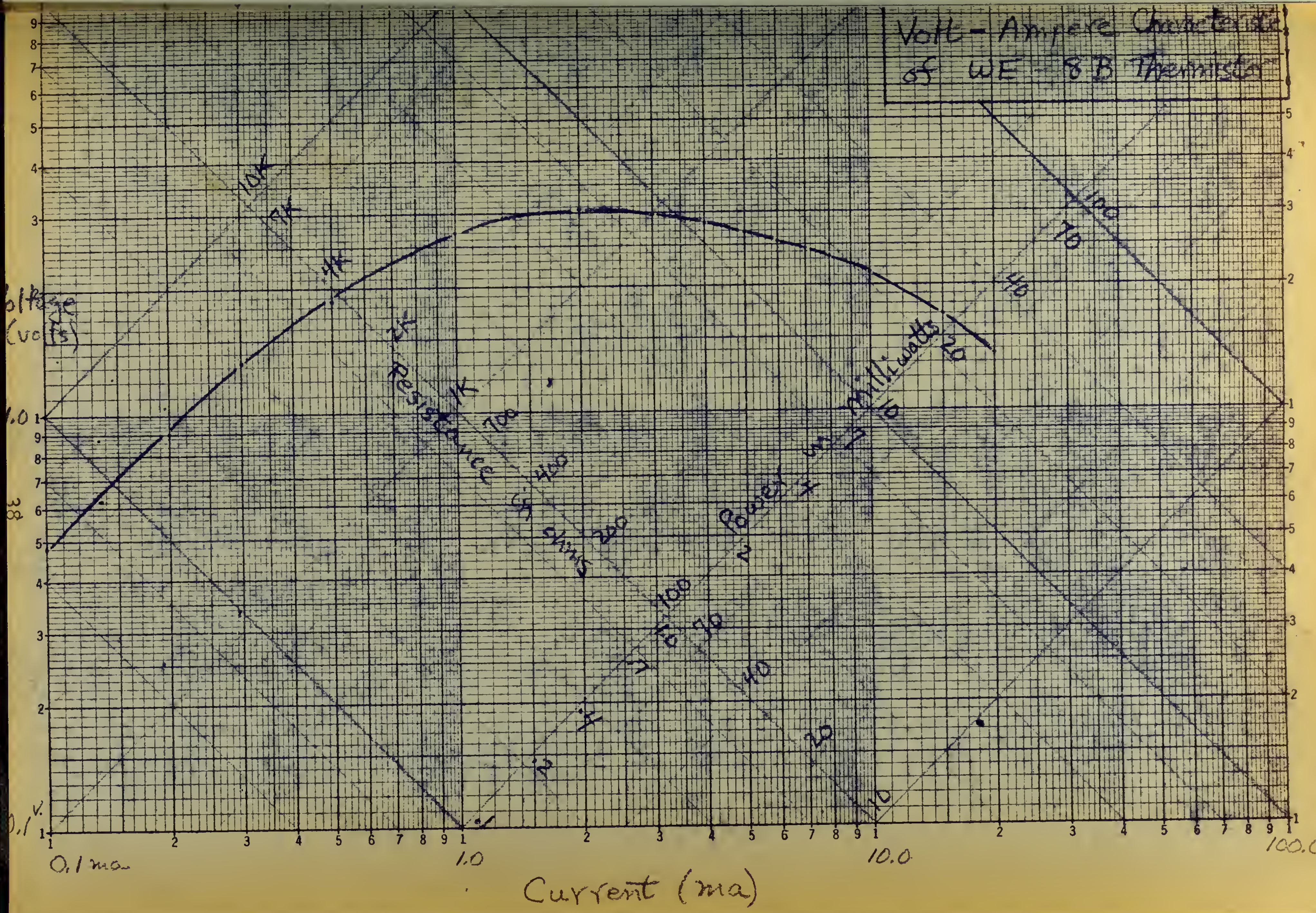
AUTOMATIC GAIN CONTROL BY VARIATIONS OF LOAD RESISTANCE

If the equivalent circuit parameters of the controlled amplifier cannot be changed without introducing excessive distortion, some variation external to the transistor becomes necessary. From the equations for the power gain of the three configurations previously given (Equations 4-1, 4-2, 4-3), it can be seen that the power gain for the common base and common emitter circuits is directly proportional to the load resistance, provided that the load resistance is much less than the quantity $r_c(1-a)$. When this inequality no longer holds as the load resistance is increased, the gain ceases to increase and even decreases slightly. This is shown clearly by the curve in Figure 11; the theoretical values for the change of insertion gain here were so close to the actual values that the plotted curves for theoretical and actual insertion gain would almost coincide.

This method of controlling gain offers some possibility of exploitation. Variable resistors of different sorts are available in a variety of resistance ranges and power ratings; the volt-ampere and temperature characteristics of a typical thermistor with which experimental work was carried out are shown in Figures 12 and 13. Note particularly



Variation
Gain with Load Resistance
4μf
7.5K
8.2K
68K
51K
10μf
1μf
130
R_ℓ
f = 60Kc
600Ω
+20
Insertion Gain (db)
+10 db
0
-10
100 Ω
1K
10K
Load Resistance (R_ℓ) (ohms)


Volt-Ampere Characteristic
of WE-8B Thermistor
Voltage (volts)
Current (ma)
0.1 ma
1.0
10.0
100.0
Resistance in ohms
10K
7K
4K
2K
1K
700
400
200
100
70
40
20
10
7
4
2
Power in Milliwatts
100
70
40
20
10
7
4
2
.1
38

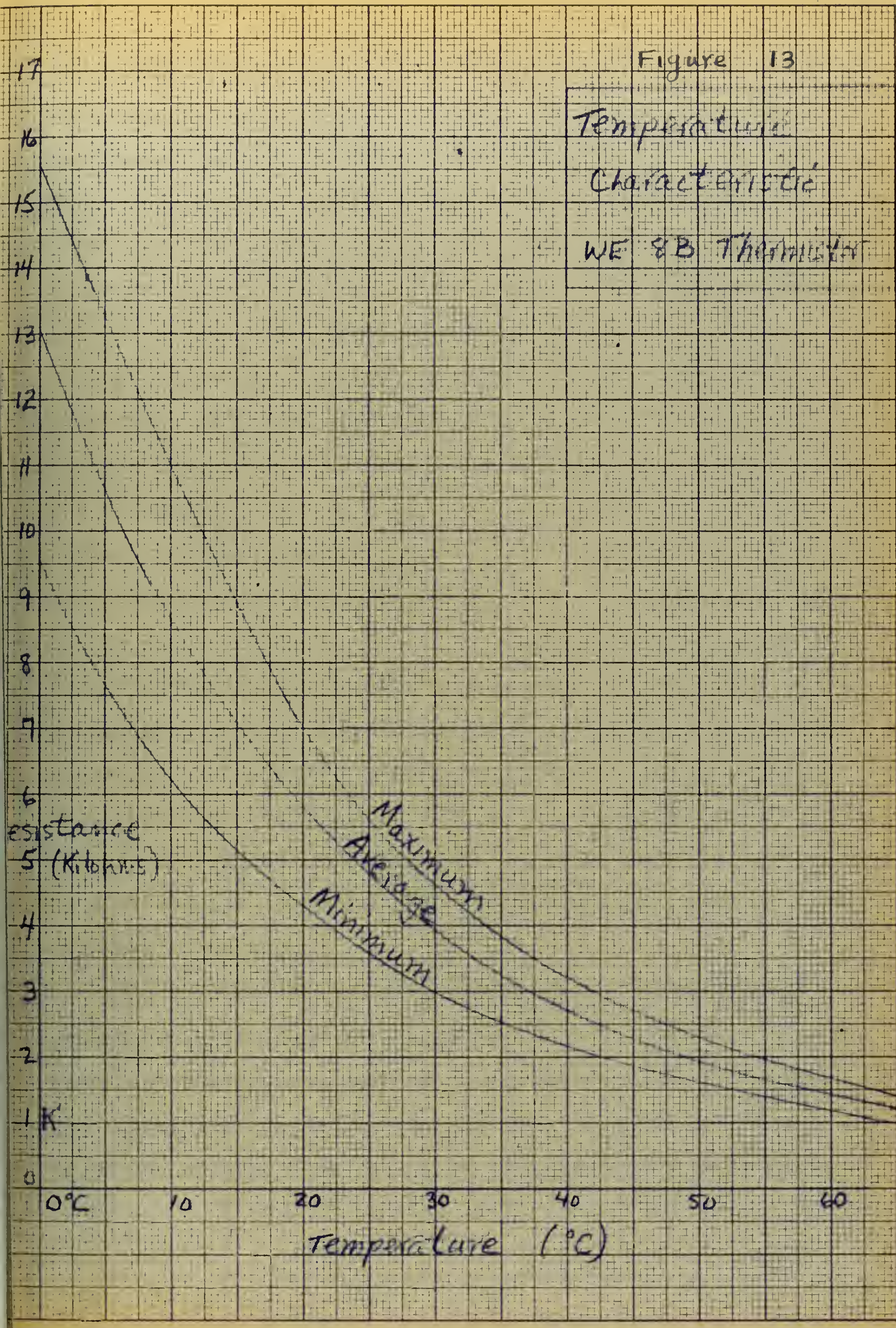
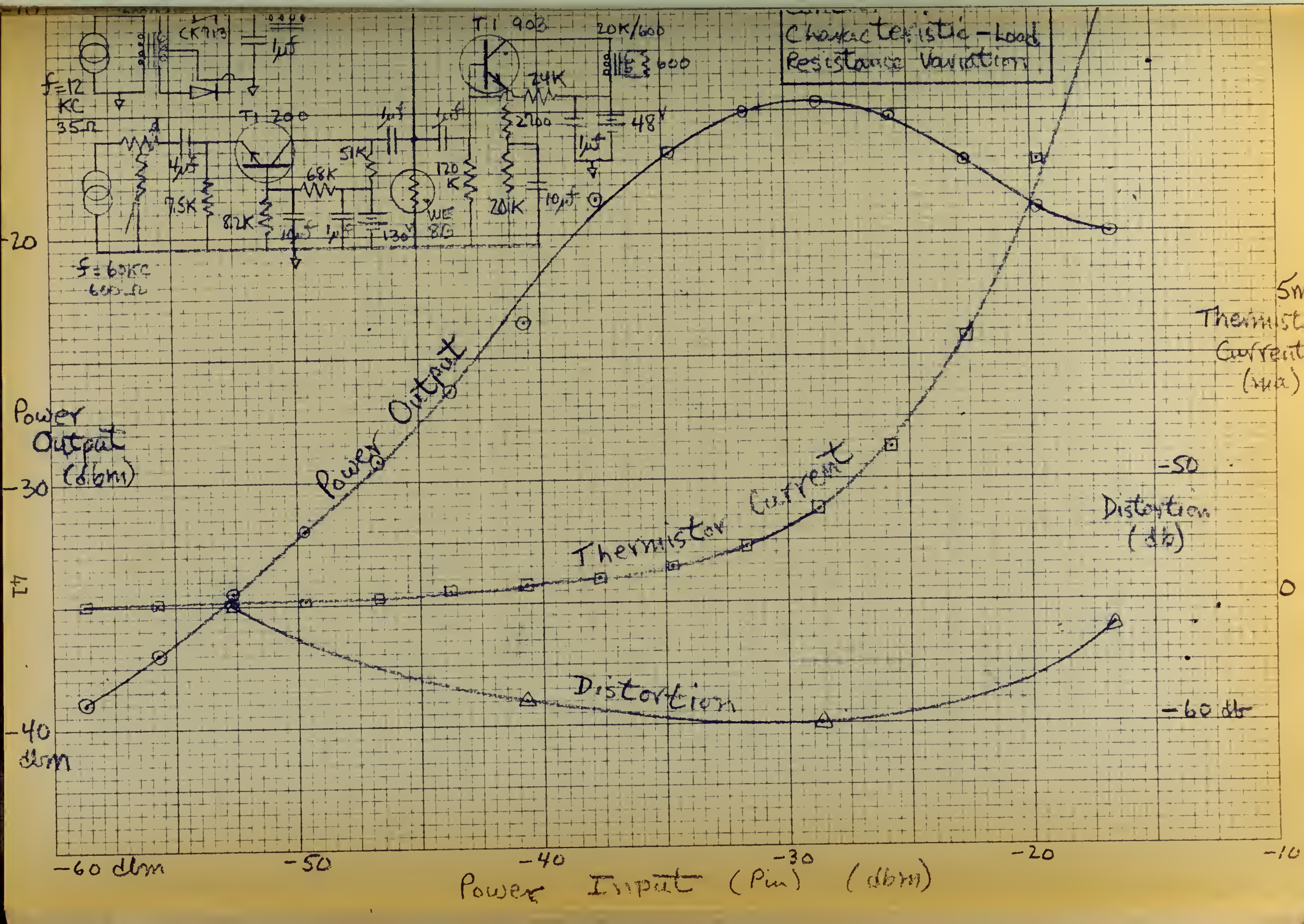


Figure 13
Temperature
Characteristic
WE 8B Thermistor
Resistance
(Kilohms)
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1 K
0
0°C
10
20
30
40
50
60
Temperature (°C)
Maximum
Average
Minimum

the rather large amount of power required to obtain low resistance ranges; this is the major disadvantage of these non-linear devices in general. From the curves of Figures 11 and 12, we can calculate that with thermistor current variations of 0.5 ma. to 10.0 ma., requiring a maximum power of 20 mw., the gain of a common base circuit could be changed about 13 db; and such a variation is actually obtained. However, when another stage is placed across the thermistor as a high impedance load, the change of gain through the circuit for a thermistor current variation of 0.5 ma. to 10.0 ma. is found to be about 22 db. This, of course, is due to the variation of the power division between the thermistor and the subsequent stage: the power applied to the second stage is times the output power of the first stage, and if the input resistance (r_i) of the subsequent stage is large compared to the thermistor resistance (R_T), the power division is directly proportional to the thermistor resistance. This would indicate a gain variation of 26 db theoretically, which is reasonably close to that actually observed when it is considered that r_i was only about $4R_{T_{max}}$.

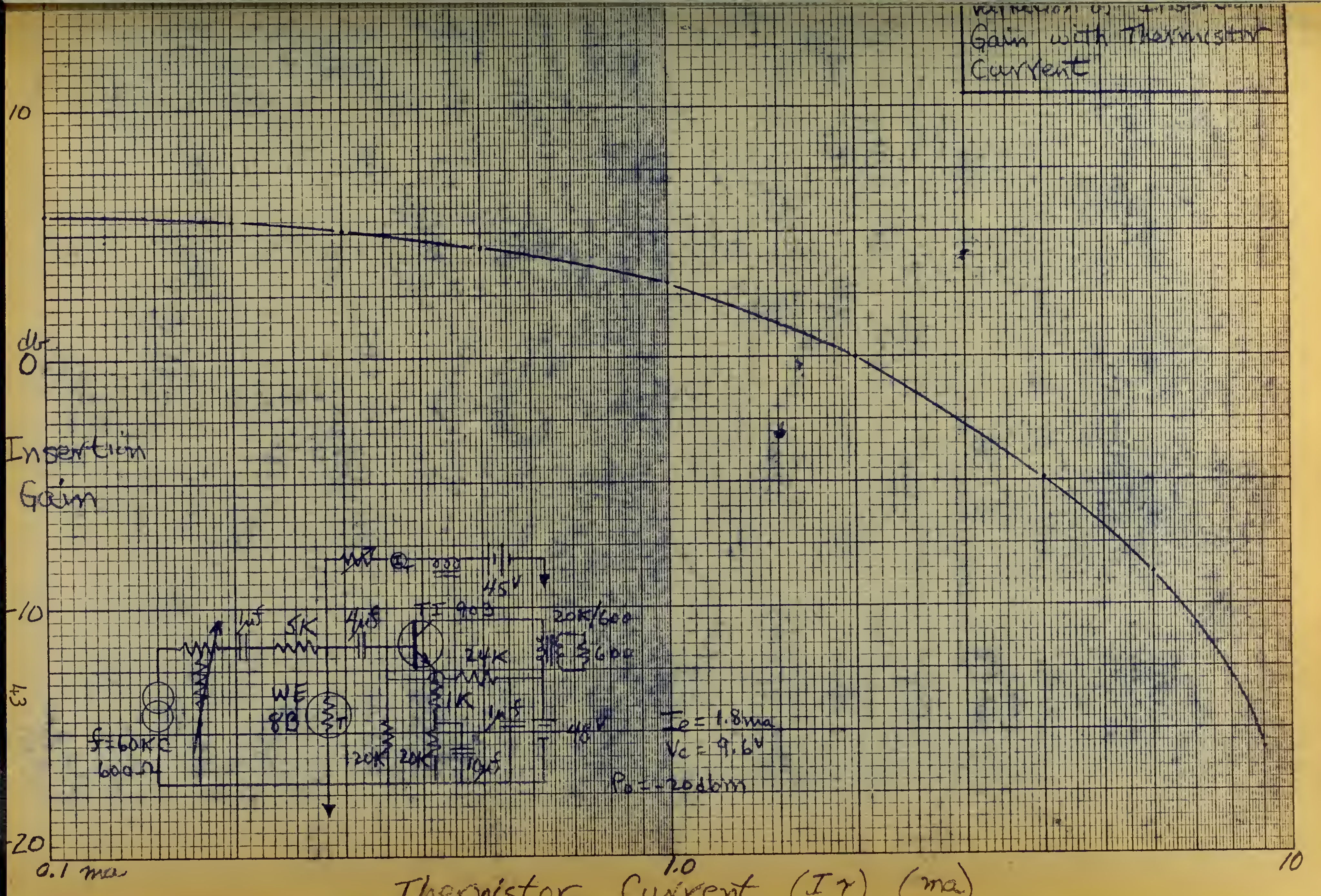
The gain control characteristic of a forward acting system employing this method is shown in Figure 14. Separate oscillators were used here to avoid measuring in the output the distortion created by the rectifier and further to avoid the construction of a high gain amplifier to furnish


Characteristic - Load
Resistance Variation
T1 903
20K/600
600
24K
2700
48
1µf
10µf
20K
120 K
WE 8G
51K
68K
T1 200
CK713
1µf
f=12 KC
35Ω
7.5K
82K
130
f = 60KC
600Ω
Power Output (dbm)
-20
-30
-40 dbm
41
Power Output
Thermistor Current
Distortion
Thermistor Current (ma)
5m
0
-50
-60 db
Distortion (db)
-60 dbm
-50
-40
-30
-20
-10
Power Input (Pin) (dbm)

the control power. The outputs of the two oscillators were changed by equal amounts for each set of readings to simulate the forward acting control system. A backward acting system would be just as effective here; also miscellaneous methods of employing the thermistors as load resistances can be used, the most obvious being a cascade arrangement, but these in general entail a much greater power requirement with high insertion losses.

The same variation of gain may be achieved by omitting the first stage and supplying the circuit from a high impedance source or a constant current generator. Here the thermistor acts as a variable load on the constant current source and further controls the power division between itself and the subsequent stage in exactly the same manner as described above. As can be seen from Figure 15, the range of gain variation obtainable is about the same as before, but a decrease in insertion gain has been traded for the advantage of eliminating one of the transistors.

Therefore the maximum control available by this method appears to be about 22-26 db, with very low distortion and high power requirements.


Variation of Insertion Gain with Thermistor Current
10
db
0
Insertion Gain
-10
-20
0.1 ma
1.0
10
Thermistor Current (Ir) (ma)
f=60KC
600Ω
WE 8B
1uf
5K
4uf
TI 903
45V
20K/600
600
24K
1K
1uf
20K
20K
10uf
48V
Ie = 1.8ma
Vc = 9.6V
Po = -20dbm
43

CHAPTER VI

AUTOMATIC GAIN CONTROL

BY VARIATIONS OF FEEDBACK RESISTANCE

Another means of controlling the gain of a transistor amplifier is by varying the feedback applied to the stage. Since distortion is to be avoided, negative feedback is the logical kind to employ. Upon consideration of the direction of the a-c current flow in a common base amplifier with unbypassed resistance between the base and ground, one can see that the output current flowing in the base produces a voltage aiding the input current flow. Although this appears to be positive feedback, the net effect of the total base current flowing in the base resistance is to produce a gain reduction, since the emitter current is greater than the collector current for junction transistors. The effect of this base resistance on input resistance, current gain and power gain can be seen from the following formulae derived in Appendix II and repeated here for convenience:

$$r_i \approx r_e + \frac{R_b\left[r_c(1-a)+n^2 R_l\right]}{r_c} \qquad (6\text{-}1)\ (II\text{-}1)$$

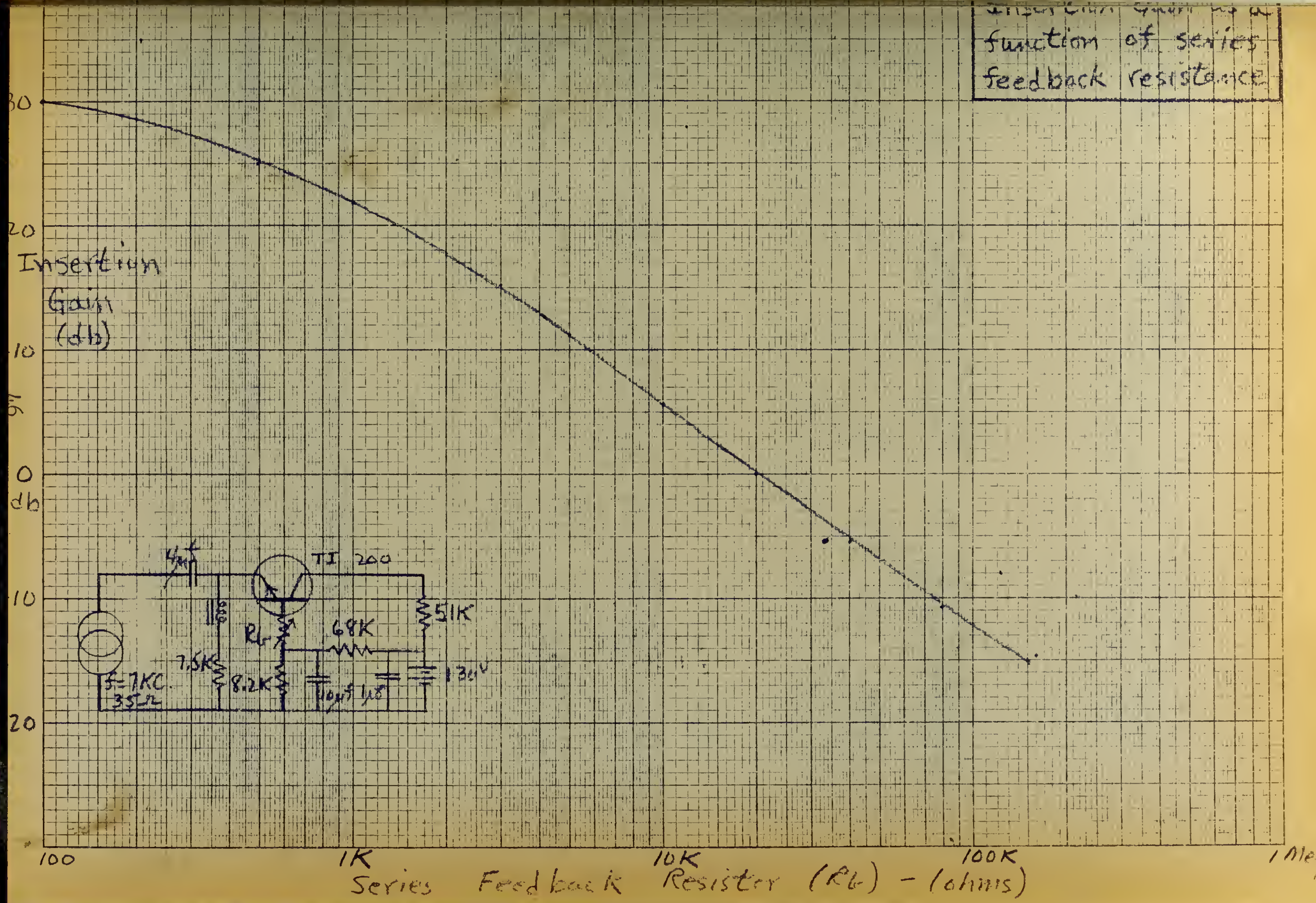
$$A_i \approx a \qquad (6\text{-}2)\ (II\text{-}3)$$

$$G \approx \frac{a^2 r_c n^2 R_l}{\left[r_c(1-a) + n^2 R_l\right]\left[\frac{r_e r_c}{r_c(1-a) + n^2 R_l} + R_b\right]} \qquad \begin{matrix}(6\text{-}3)\\(\text{II-}5)\end{matrix}$$

Due to the output and input currents flowing in the opposite directions in the base resistance, it would be expected that a large resistance would have to be added at this point to achieve a given amount of feedback. As can be seen from Figure 16, this was borne out experimentally, rather large feedback resistances being required to achieve a moderate degree of feedback. This circuit does not appear to offer much promise as an automatic gain control circuit, particularly compared to the range of variation obtainable with small values of feedback resistance in the common emitter circuit.

Due to the flow of the output and input currents in the same direction through the emitter resistance in a common emitter circuit, the net feedback should be much greater for a given feedback resistance. The formulae for input resistance, current amplification, and power gain, also derived in Appendix II, are given below:

$$r_i \approx r_b + \frac{r_c}{1 + \frac{r_c(1-a) + n^2 R_l}{R_e + r_e}} \qquad \begin{matrix}(6\text{-}4)\\(\text{II-}13)\end{matrix}$$



Insertion Gain as a function of series feedback resistance
30
20
10
0
db
10
20
Insertion Gain (db)
TI 200
51K
68K
Rb
7.5K
8.2K
f=1KC
35Ω
130V
100
1K
10K
100K
1 Meg
Series Feedback Resister (Rb) - (ohms)

$$A_i \approx \frac{a\,r_c}{r_c(1-a) + n^2 R_\ell + R_\epsilon} \qquad \begin{matrix}(6\text{-}5)\\(II\text{-}15)\end{matrix}$$

$$G \approx \frac{a^2 r_c^2 n^2 R_\ell}{\left[r_c(1-a)+n^2R_\ell+R_\epsilon\right]\left[r_c\{R_\epsilon+r_\epsilon+r_b(1-a)\}+r_b n^2 R_\ell\right]} \qquad \begin{matrix}(6\text{-}6)\\(II\text{-}17)\end{matrix}$$

Note that for Re $\gg r_e$ and Re $\gg r_b$,

$$G \approx \frac{a^2 r_c n^2 R_\ell}{R_\epsilon^2\left[1 + \frac{r_c(1-a)+n^2R_\ell}{R_\epsilon}\right]} \qquad (6\text{-}7)$$

and as Re becomes $\gg r_c(1-a) + n^2R_1$,

$$G \approx \frac{a^2 r_c n^2 R_\ell}{R_\epsilon^2} \qquad (6\text{-}8)$$

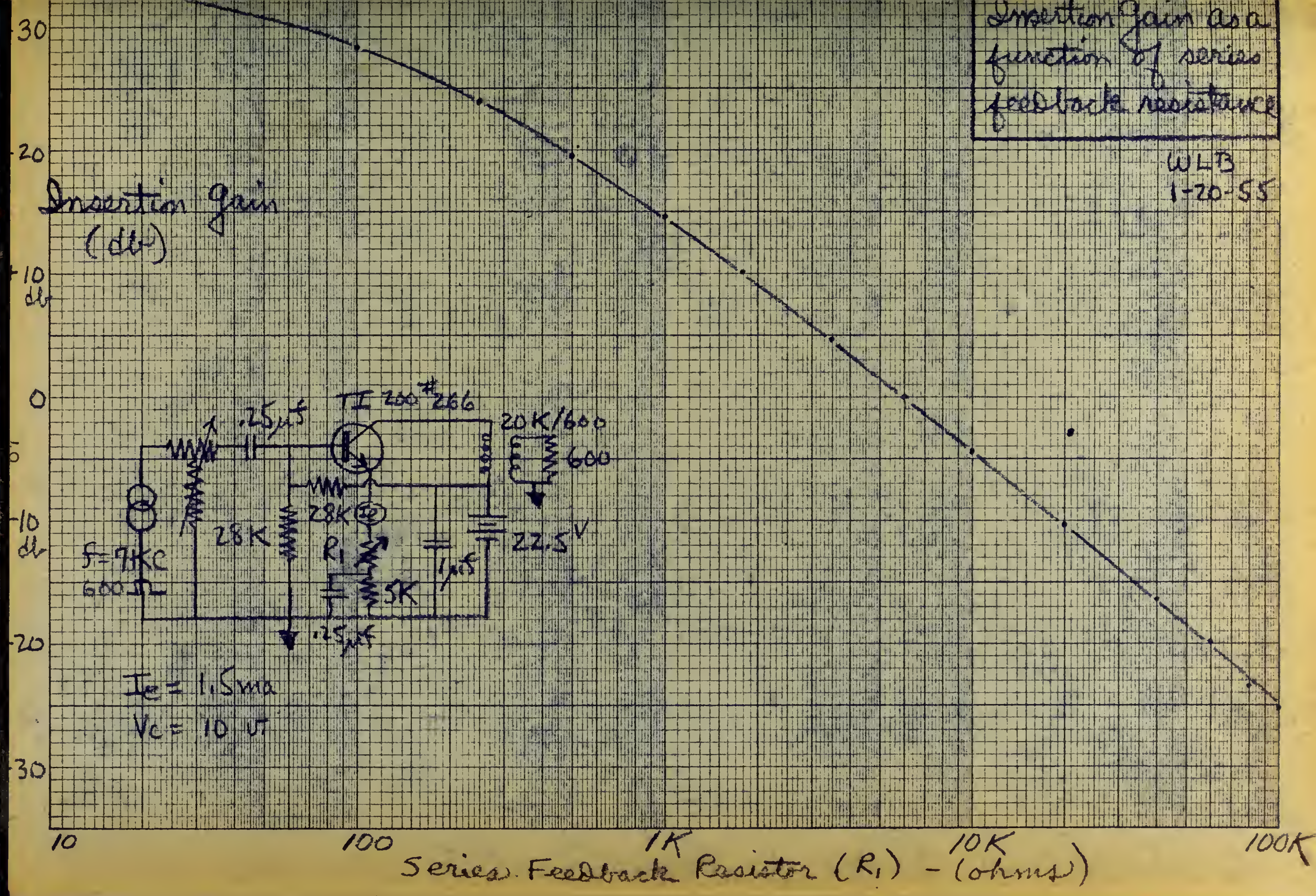
and a power gain versus emitter feedback resistance curve has a -6 db/octave slope. Notice also that under these conditions, $r_i \approx r_b + r_e \approx r_e$ and $L_i \approx \frac{4R_g}{r_c}$, a very small figure. Since the input resistance is approximately constant when Re becomes this large, there is no further increase in the input mismatch loss after Re increases past this point.

The results of tests on common emitter circuits with

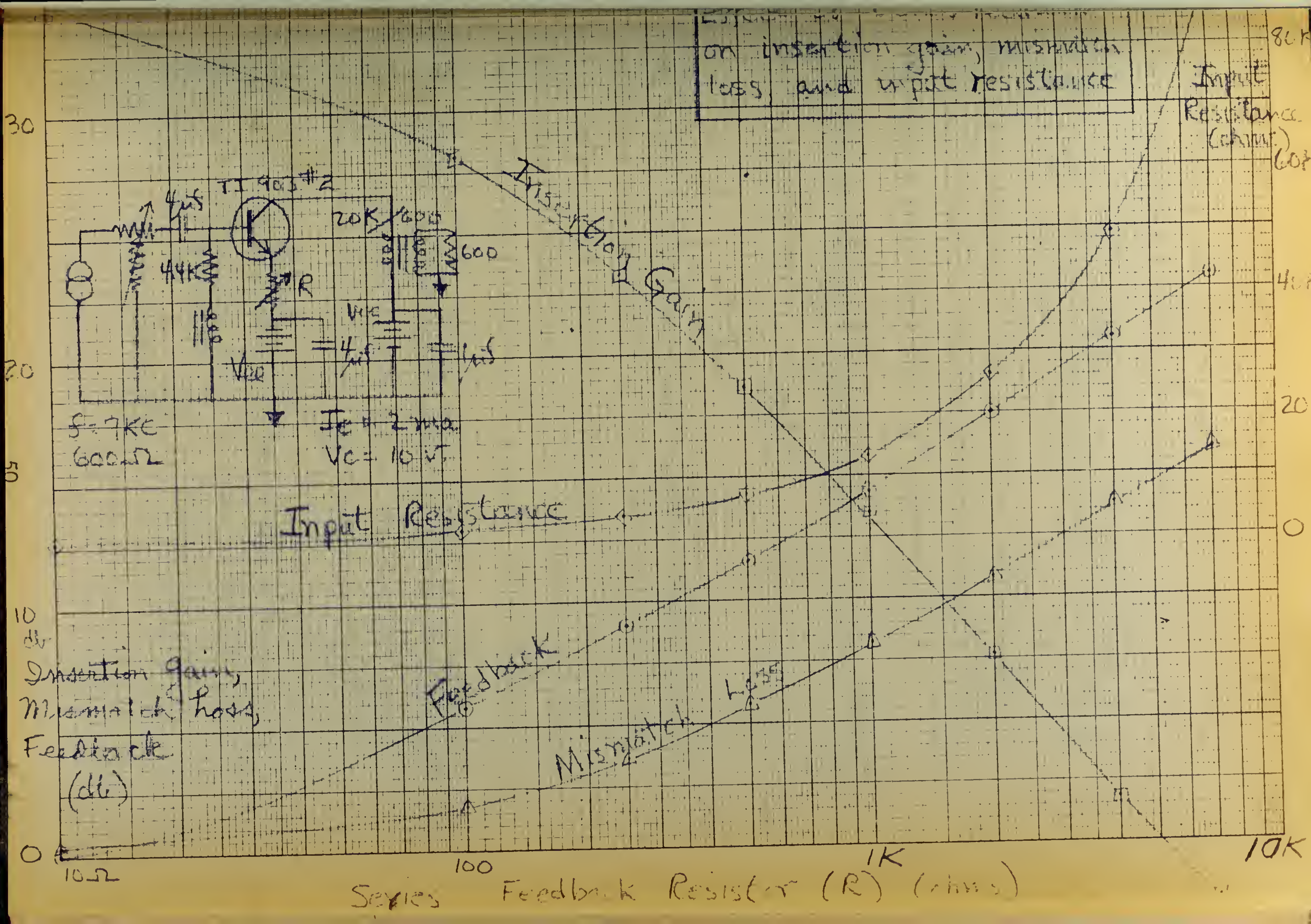
series feedback clearly show a wide range of variation of insertion gain covering more than 60 db; from Figure 17, the -6 db/octave slope can be clearly seen for large values of the series feedback resistance. For values of this resistance greater than $r_c(1-a)$ and R_1, the circuit shown acts as an attenuator; thus, although the region of the curve where the slope is greatest would be the preferable region of operation from a gain control viewpoint, attention will be focused on the lower values of the feedback resistance in the region where the circuit still provides some amplification. Through the use of additional circuitry, the steepness of the slope can be increased in this region.

An investigation of the characteristics of this circuit configuration for the feedback resistance varying up to seven kilohms (beyond which point the insertion gain becomes negative) shows that the decrease in gain with increasing feedback resistance is only slightly greater than that due to the increasing degree of mismatch at the input. (Figure 18) For Re greater than 50 ohms, the decrease in gain due to the feedback is seen to be about 3 db/octave; this figure can be confirmed by noting from equation (6-7), in the region where Re is still much smaller than the sum of $r_c(1-a)$ and R_1, the power gain is approximately

$$G \approx \frac{a^2 r_c n^2 R_\ell}{R_E\left[r_c(1-a) + n^2 R_\ell\right]} \qquad (6\text{-}9)$$


Insertion Gain as a function of series feedback resistance
WLB
1-20-55
Insertion Gain (db)
30
20
10 db
0
-10 db
-20
-30
TI 200 #266
.25 µf
20K/600
600
28K
28K
R1
5K
1µf
22.5V
f=7KC
600 Ω
.25µf
Ie = 1.5ma
Vc = 10 v.
10
100
1K
10K
100K
Series Feedback Resistor (R1) - (ohms)

on insertion gain, mismatch loss, and input resistance



and will still have a -3 db/octave slope. We note also that, from equation (6-4) for these values of Re,

$$r_i \approx \frac{r_c R_E}{r_c(1-a) + n^2 R_\ell} \quad ; \qquad (6\text{-}10)$$

the input resistance of the circuit is seen therefore to be almost linear for values of Re in this region. Since the input mismatch loss given in equation (4-7) is

$$L_i \approx \frac{4 R_g r_i}{(R_g + r_i)^2} \qquad (6\text{-}11)$$

and in this region, $r_i \gg R_g$, then

$$L_i \approx \frac{4 R_g}{r_i} \qquad (6\text{-}12)$$

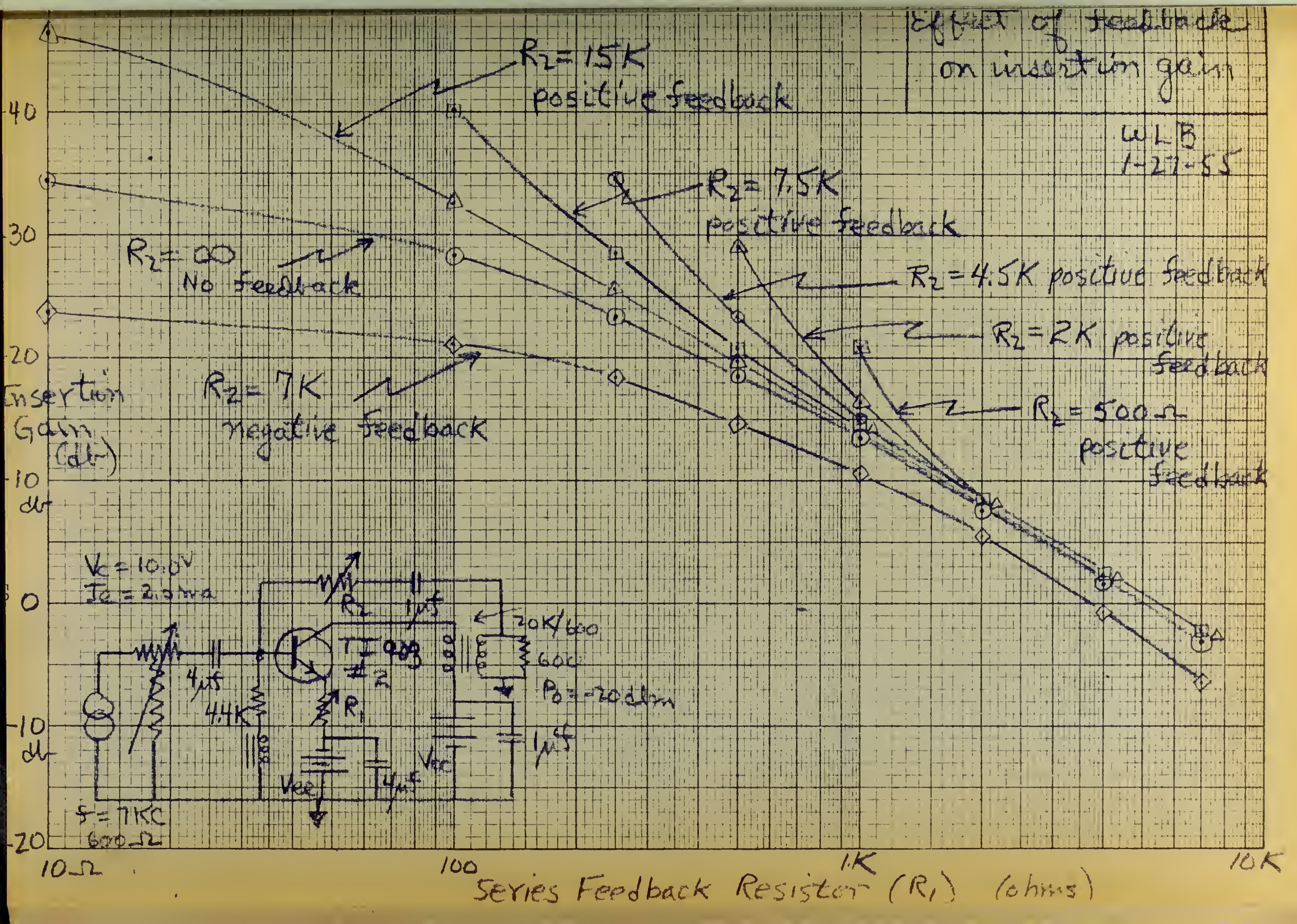
and L_i has a -3 db/octave slope as Re is increased, a fact confirmed from inspection of the curve on Figure 18.

As mentioned previously, the slope of the insertion gain versus feedback resistance can be increased in this region where Re is less than 7 kilohms. This is accomplished through the use of positive shunt feedback. That this is true can be seen by investigating the formula for the power gain for the common emitter circuit with negative series and positive shunt feedback. We desire that $\frac{\partial G(R_E, R_S)}{\partial R_E}$ be a maximum. This derivative is itself a function of Re and Rs, and we wish to observe the effect of Rs on it. Thus we find $\frac{\partial}{\partial R_S}\left(\frac{\partial G}{\partial R_E}\right)$. As shown in Appendix III, the expression for $\frac{\partial}{\partial R_S}\left(\frac{\partial G}{\partial R_E}\right)$ is always negative as

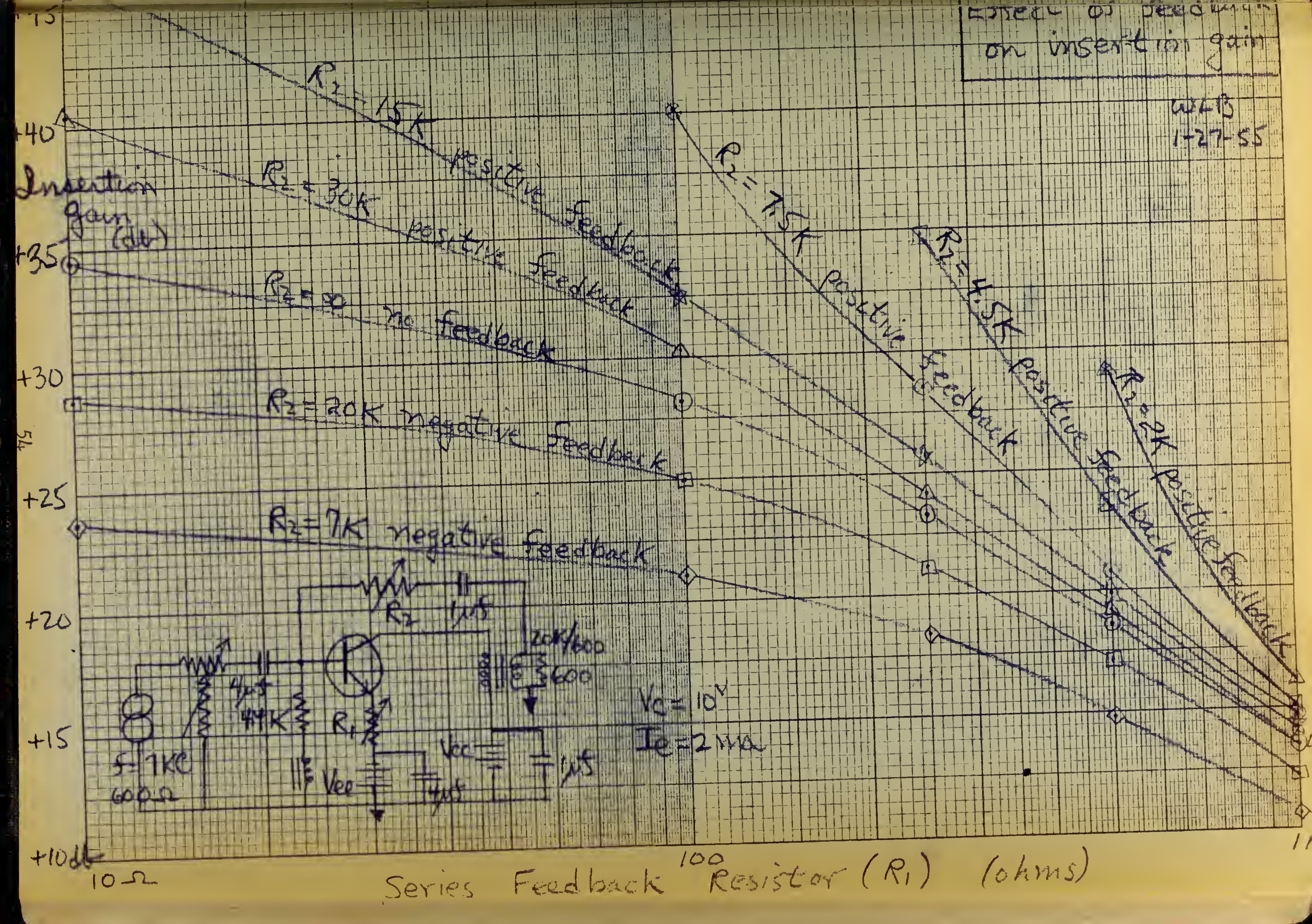
long as $r_i \gg 0$, that is, as the positive shunt feedback is increased by decreasing Rs, the rate of change of gain with increasing Re (and thus the negative series feedback) is increased. Therefore by the addition of positive shunt feedback, the change of gain obtainable for a given change of the series feedback resistance should be increased; negative shunt feedback should have the opposite effect. The results of tests on a common emitter circuit having both shunt and series feedback is shown in Figures 19 and 20. The curves can be seen to substantiate the above derivation, and the addition of positive shunt feedback can be seen to extend the range of gain control obtainable for a given series resistance variation.

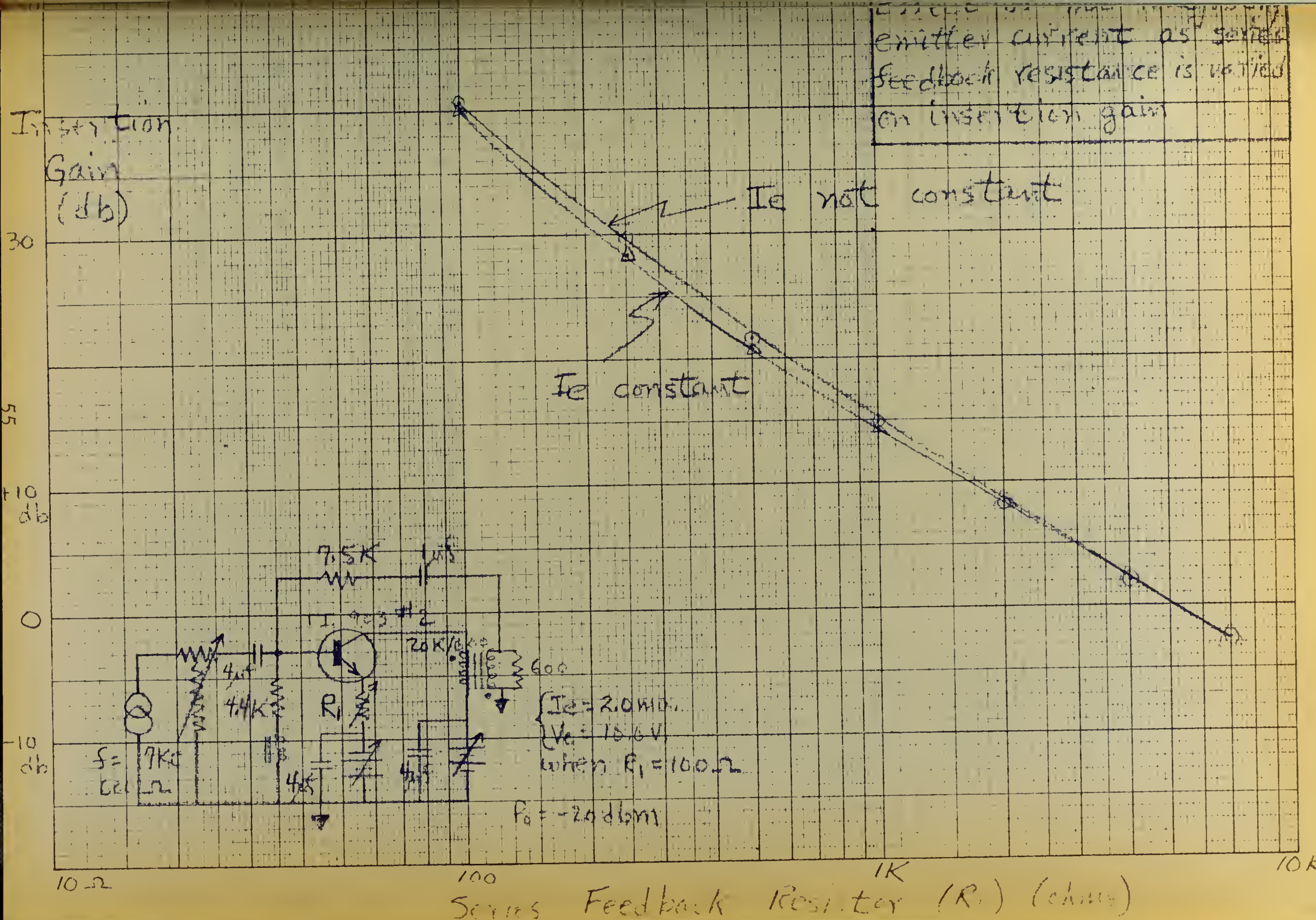
In varying the series feedback resistance, it is immaterial on insertion gain if the emitter current is maintained constant or not, as Figure 21 demonstrates. However, for large values of this resistance, when Ie becomes quite small, the distortion can be expected to increase.

The most obvious disadvantage of this system of gain control is the fact that in general positive feedback tends to decrease the bandwidth of an amplifier and to increase its distortion. However, as can be seen from Figures 19 and 20, the net resultant of the positive and negative feedback is to produce a gain reduction as soon as a small



Effect of feedback on insertion gain
WLB
1-27-55
R_2 = 15K positive feedback
R_2 = 7.5K positive feedback
R_2 = 4.5K positive feedback
R_2 = 2K positive feedback
R_2 = 500 Ω positive feedback
R_2 = ∞ No feedback
R_2 = 7K negative feedback
Insertion Gain (db)
40
30
20
10 db
0
-10 db
-20
10 Ω
100
1K
10K
Series Feedback Resistor (R_1) (ohms)
V_c = 10.0V
I_c = 2.0 ma
R_2
1 μf
20K/600
600
P_o = -20 dbm
4 μf
4.4K
R_1
#2
V_{ee}
V_{cc}
4 μf
1 μf
f = 1KC
600 Ω


Effect of feedback on insertion gain
WLB
1-27-55
+45
+40
+35
+30
+25
+20
+15
+10db
Insertion gain (db)
R_2 = 15K positive feedback
R_2 = 30K positive feedback
R_2 = ∞ no feedback
R_2 = 20K negative feedback
R_2 = 7K negative feedback
R_2 = 7.5K positive feedback
R_2 = 4.5K positive feedback
R_2 = 2K positive feedback
f = 1KC
600 Ω
4μf
47K
R_1
R_2
1μf
Vee
4μf
Vcc
1μf
20K/600
600
Vc = 10V
Ie = 2 ma
10 Ω
100
Series Feedback Resistor (R_1) (ohms)



emitter current as series
feedback resistance is varied
on insertion gain
Insertion
Gain
(db)
30
+10 db
0
-10 db
Ie not constant
Ie constant
7.5K
4.4K
R_1
600
I_e = 2.0 ma.
when R_1 = 100 Ω
P_o = -20 dbm
f = 1Kc
1K Ω
10 Ω
100
1K
10K
Series Feedback Resistor (R_1) (ohms)

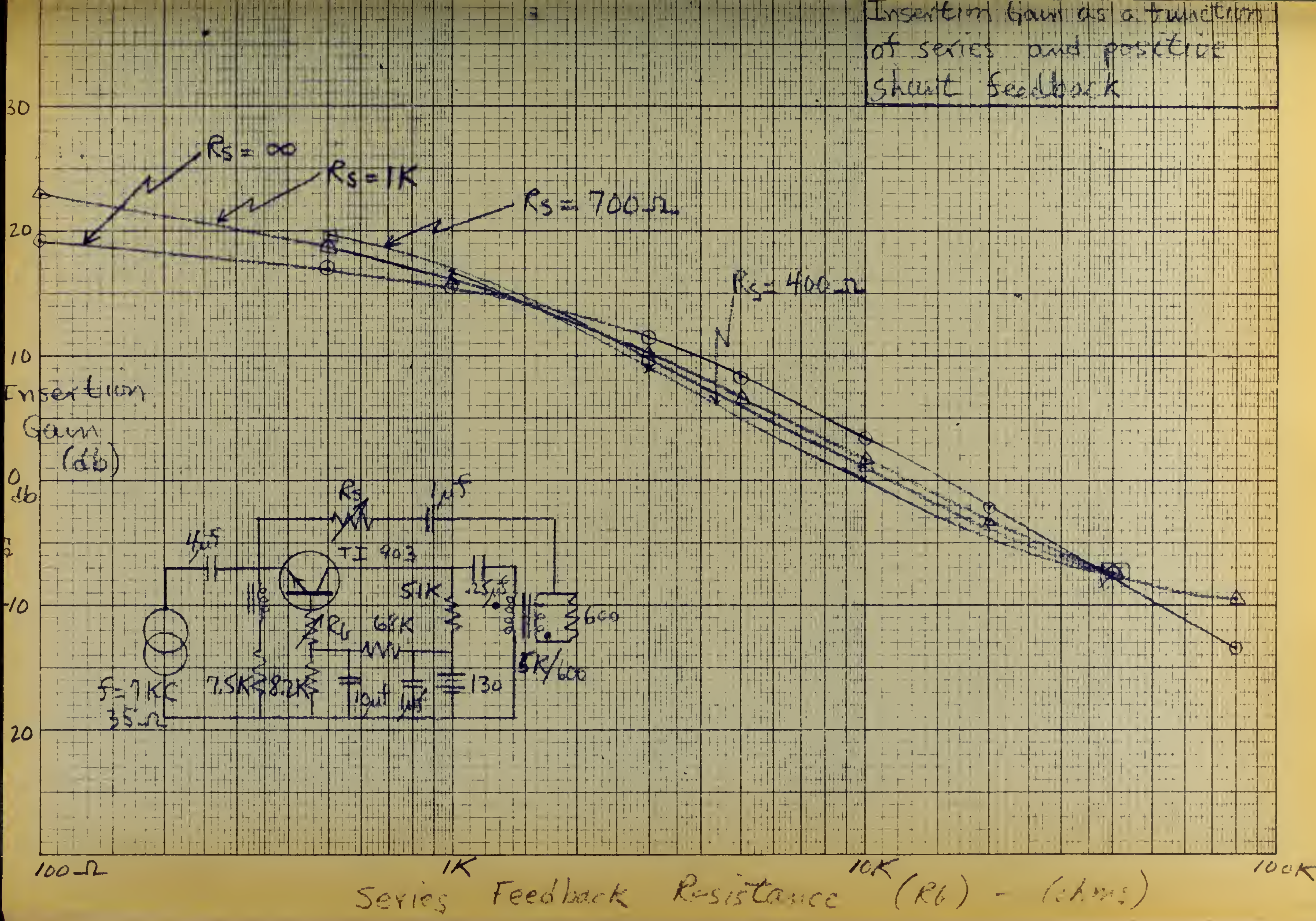
amount of resistance is added in the emitter leg. Therefore it is to be expected that the higher distortion and decreased bandwidth is only to be expected for the lowest value of series emitter resistance. Data taken proves that the addition of positive feedback does not appreciably affect the distortion provided that the series feedback resistance is limited to values somewhat greater than its minimum value. The frequency response of the amplifier will also be deteriorated for the lowest value of series feedback resistance due to the positive feedback, but this will not be serious if the same restriction as is necessary to prevent excessive distortion is maintained.

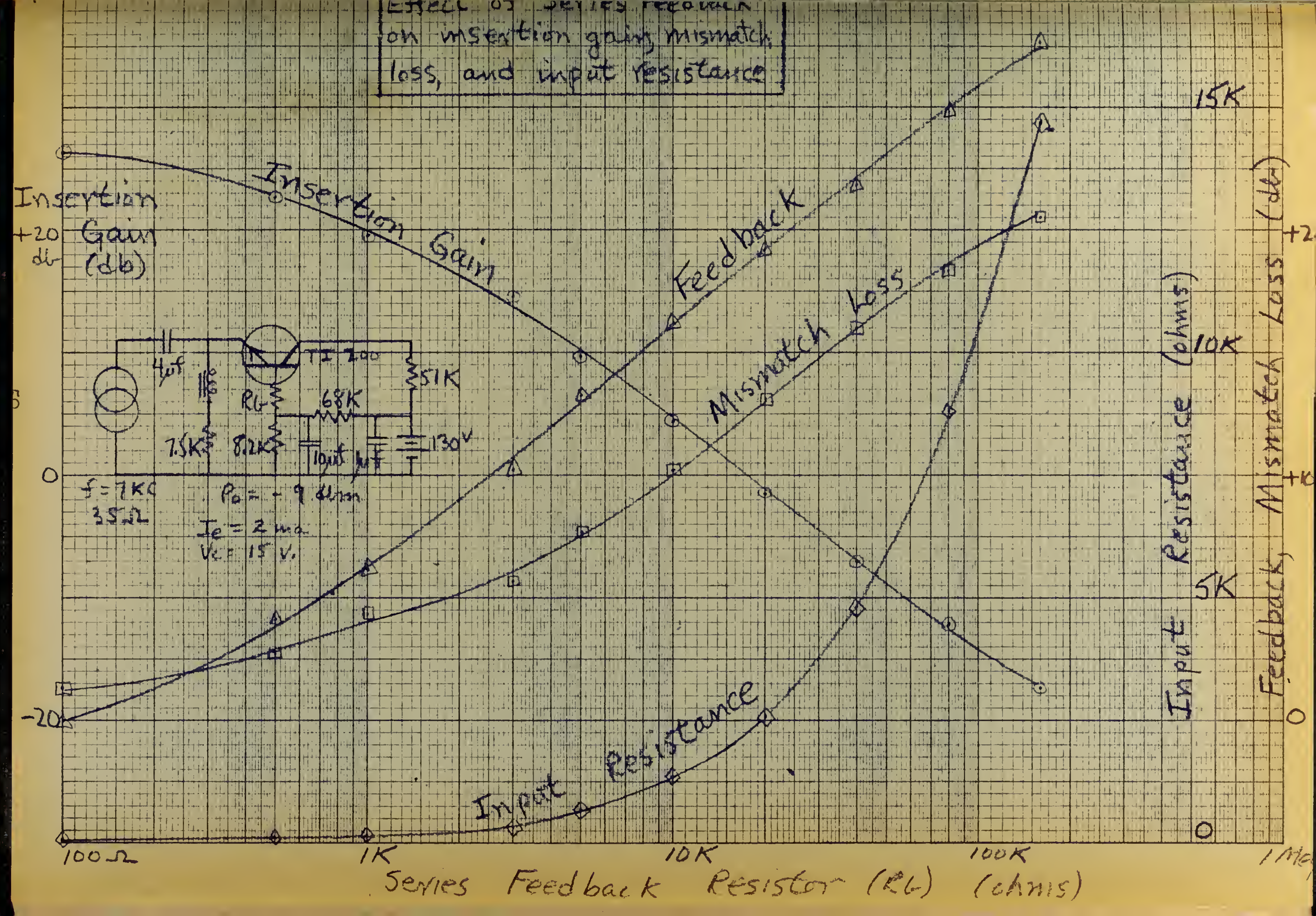
Of course this system would be difficult to apply to a wide band amplifier having an upper frequency range that began to approach the power gain cutoff frequency of the transistor being used. For the common emitter configuration this may occur at frequencies very much lower than the alpha cutoff frequency of the device. Further the phase distortion produced in the amplifier in this range makes the application of feedback over a band of frequencies a difficult accomplishment, the result of the phase shift with frequency being a rapid deterioration of the frequency response of the amplifier. However, for a single frequency amplifier where a phase correcting network can be added in the shunt feedback path and where there is no

concern for frequency response over a band of frequencies, this system offers a wide range of control, up to 40 db with a maximum distortion of -46 db for a series emitter resistance variation of only 100 ohms to 2000 ohms.

It might appear that for applications where a wide bandwidth is required a common base circuit in which the feedback is varied might be used in spite of the decreased amount of gain control we find in such a configuration. Here again we might expect the addition of positive shunt feedback to increase our range of gain control as was found to be true for the common emitter circuit. The results of tests run to ascertain if this was actually true indicate that the addition of positive shunt feedback does increase the slope of our gain-series feedback resistance curve out to a value of Rb of about ten kilohms. (Figure 22) However, the effect is seen to be quite small and not worth the increased complexity, increased distortion, and reduced bandwidth also obtained with such feedback.

An analysis of the decrease in gain due to feedback and due to the mismatch loss at the input shows results that are extremely similar to those obtained in the similar analysis of the common emitter circuit with series feedback; this is reasonable and predictable, since the various slopes observed can be derived in a manner similar to those derived previously for the common emitter circuit, and are found to be almost identical. (Figure 23)

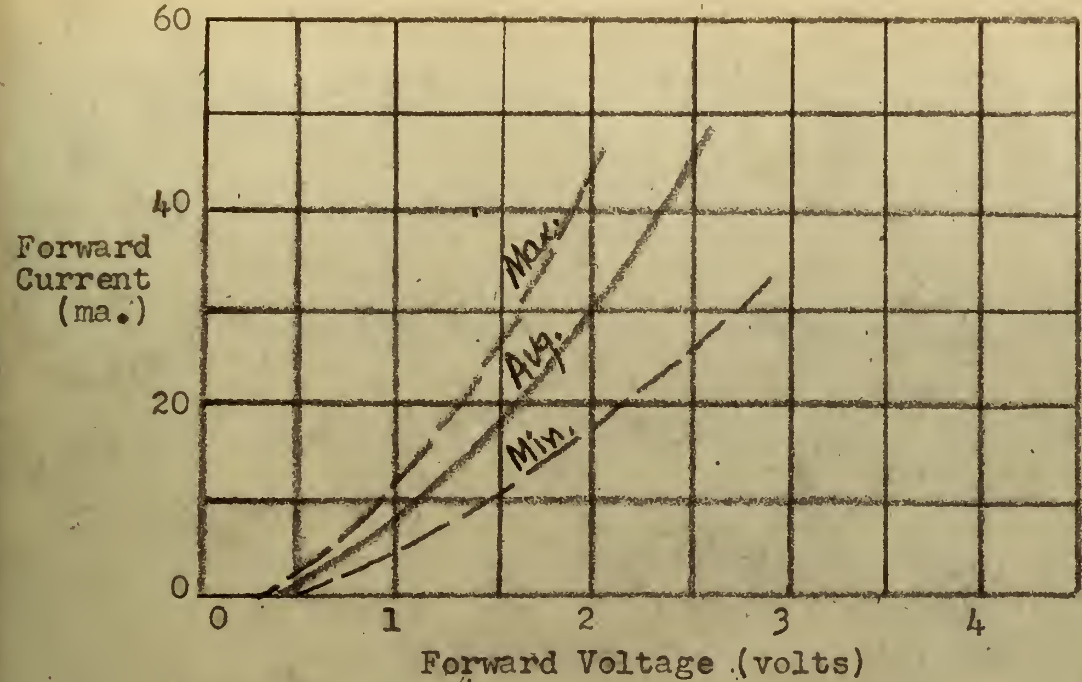

Insertion Gain as a function of series and positive shunt feedback
$R_s = \infty$
$R_s = 1K$
$R_s = 700\Omega$
$R_s = 400\Omega$
30
20
10
Insertion Gain (db)
0 db
-10
20
TI 903
R_s
R_L
1μf
4μf
51K
68K
7.5K
8.2K
10μf
130
600
5K/600
f = 1KC
35Ω
100Ω
1K
10K
100K
Series Feedback Resistance (R_L) - (ohms)



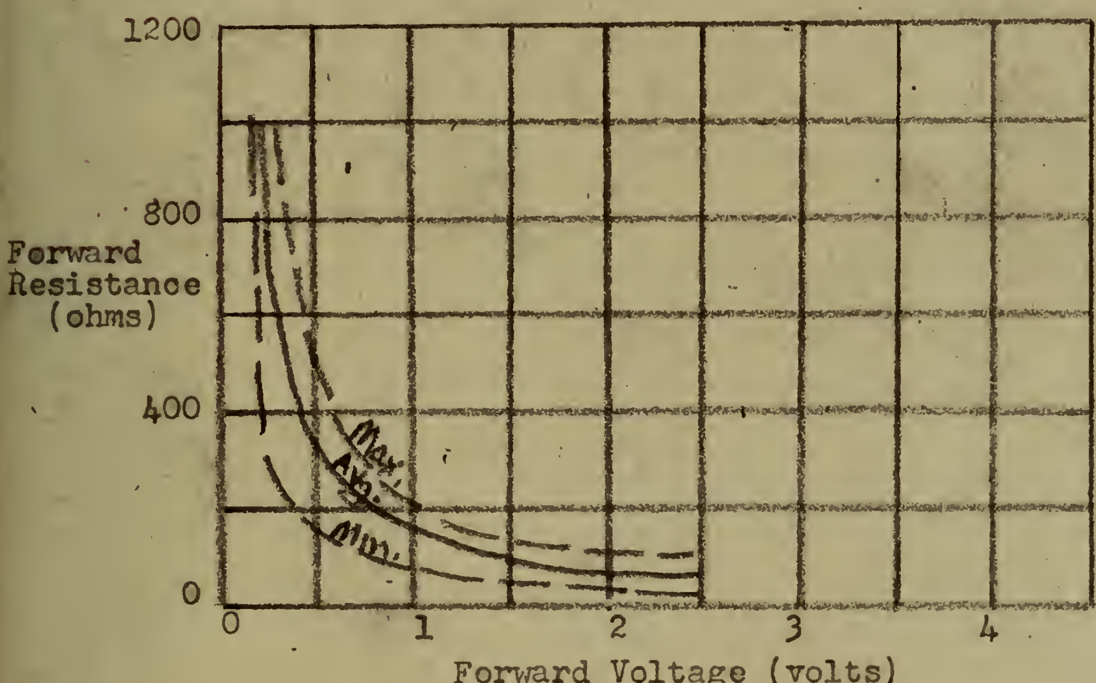
on insertion gain, mismatch loss, and input resistance
Insertion Gain
Feedback
Mismatch Loss
Input Resistance
Insertion Gain (db)
+20 db
0
-20
15K
10K
5K
0
Input Resistance (ohms)
Feedback, Mismatch Loss (db)
4μf
7.5K
8.2K
RL
TI 200
68K
51K
10μf
1μf
130V
f = 7KC
35Ω
Ie = 2 ma
Vc = 15 V.
100Ω
1K
10K
100K
Series Feedback Resistor (RL) (ohms)

Even without the application of positive feedback, this circuit offers some advantage over the common emitter circuit, due to the fact that its power gain cutoff frequency is higher. However, the results of tests show that for medium values of series feedback resistance there is deterioration of frequency response and an increase in distortion. With higher values of series feedback resistance the frequency response improves and the distortion decreases. This effect is most likely caused by the positive feedback effect of the unbypassed base resistance. Because of this deterioration of frequency response (the distortion is not limiting here and is below -40 db for power outputs of -30 dbm) and the lesser range of gain variation obtainable, the common base circuit is considered to be inferior to the common emitter.

Having analyzed the effect of series feedback on controlling the gain of a common emitter amplifier, it still remains to obtain a variable resistance with a range as great as possible and yet within an appropriate range of values that permit the operation of the amplifier with a net gain (otherwise we have in effect only a complex attenuator). From inspection of the values of resistance required, it might be decided that they could be obtained readily from a forward biased semiconductor diode. A typical E-I characteristic for a germanium diode is shown in Figure 24, along with its corresponding resistance variation with current. Figure 25 shows the corresponding forward current

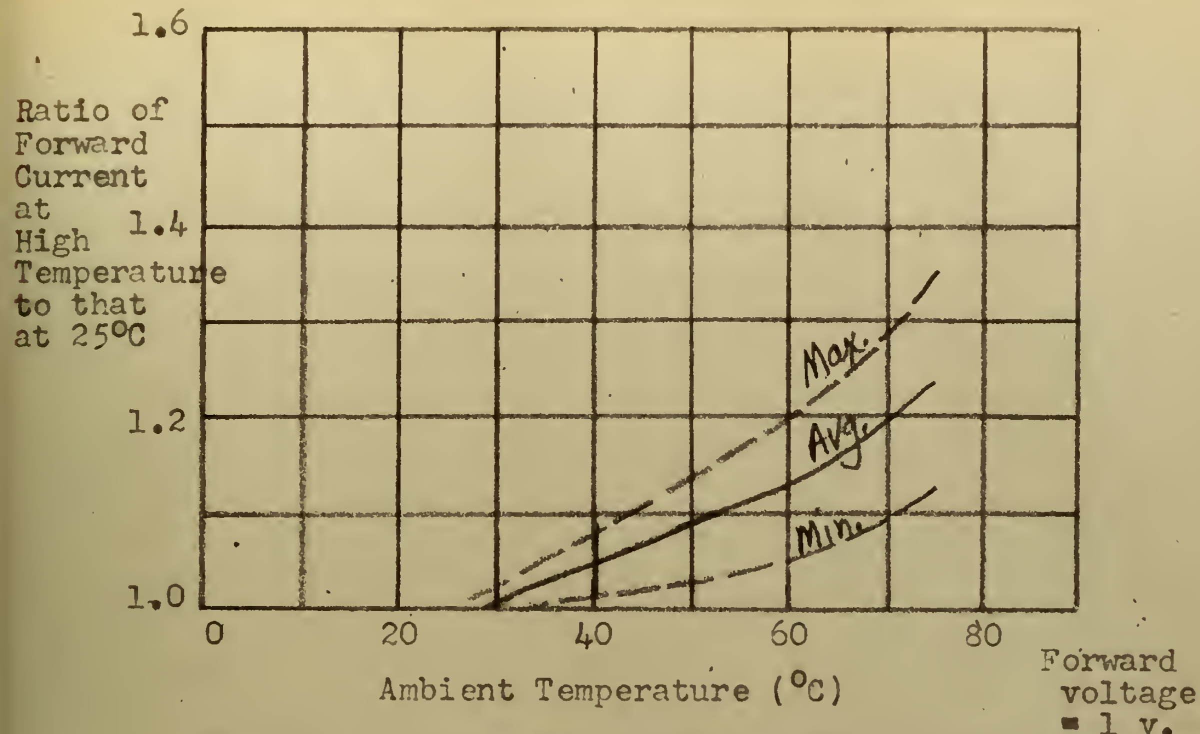


TYPICAL DIODE STATIC FORWARD CURRENT-VOLTAGE CHARACTERISTIC (25°C)

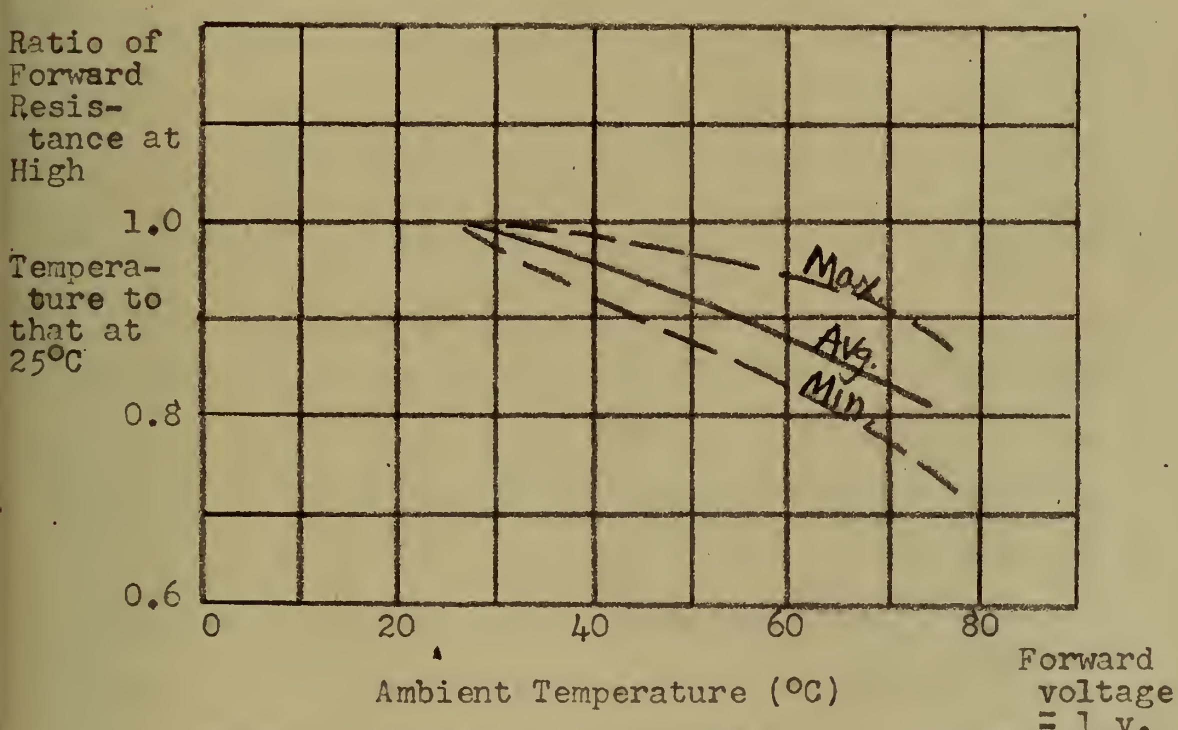


TYPICAL DIODE STATIC FORWARD RESISTANCE-VOLTAGE CHARACTERISTIC (25°C)

Figure 24



TYPICAL DIODE FORWARD CURRENT-TEMPERATURE RATIO CHARACTERISTIC WITH RESPECT TO 25°C



TYPICAL DIODE FORWARD RESISTANCE RATIO-TEMPERATURE CHARACTERISTIC WITH RESPECT TO 25°C

Figure 25

and resistance variations with temperature changes. Note the wide range of variation possible between individual units and the dependence of the characteristic upon temperature. These possible variations between individual units coupled with the distortion which would be introduced by the device when operated at low d-c currents are disqualifications for the use of diodes to accomplish the desired variable resistance characteristic.

As already mentioned, the thermistor also presents a variable resistance in the desired range of variation. Theoretically with a thermistor current variation of between 0.5 ma. and 10.0 ma., a gain change of 19 db can be obtained when the thermistor is employed as the series feedback resistance in a common emitter circuit. This is with no positive feedback applied; with it, an even greater change of gain is obtainable. Note that with this system the thermistor current must decrease with an increase of the input signal in order that the feedback resistance increase, thus reducing the amplifier gain.

One of the chief disadvantages of using a thermistor as the variable resistance element is the large power consumption required for its operation. As previously mentioned, this can require up to 20 mw. In a search for a lower powered device, recourse can be had to a transistor itself, since we have seen that for varying operating points the input and output impedances of the various configurations all change. Since the gain of the stage varies

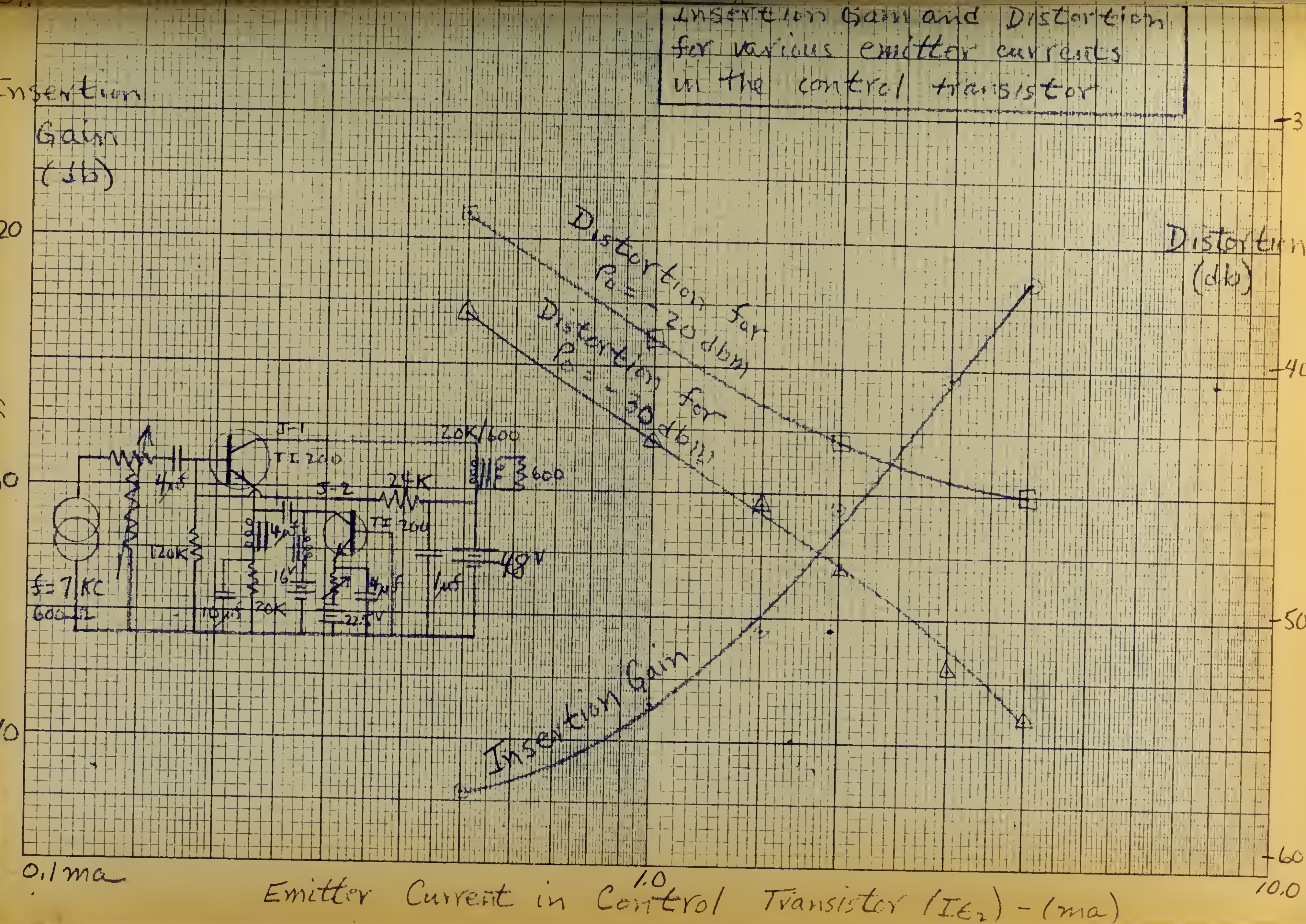
approximately inversely with the logarithm of the series resistance, for maximum gain change we desire that the ratio of the maximum resistance to the minimum resistance be as great as possible. From a study of the input resistance of the three basic configurations for the output shorted, open circuited or with a load resistance approximately equal to the collector resistance, and of the output resistance for source impedances of the same values, using the equivalent circuit parameters obtained in Test D, Appendix I (where the emitter current is kept high enough to obviate distortion) as being fairly typical, the following circuits presented the highest ratio of maximum to minimum resistance obtainable:

Common emitter, output resistance, $R_g = 0$:	for $I_e = 4$ ma.	66K
	for $I_e = 0.5$ ma.	516K
	Ratio:	7.8
Common base, input resistance, $R_1 = 0$, and common collector, output resistance, $R_g = 0$:		
	for $I_e = 4$ ma.	9 Ω
	for $I_e = 0.5$ ma.	68 Ω
	Ratio:	7.56
Common base, output resistance, $R_g = 0$:	for $I_e = 4$ ma.	70K
	for $I_e = 0.5$ ma.	527K
	Ratio:	7.53

All other combinations yielded ratios less than 5.00. Note that the necessary a-c conditions for generator or load impedances can be achieved easily in the first three

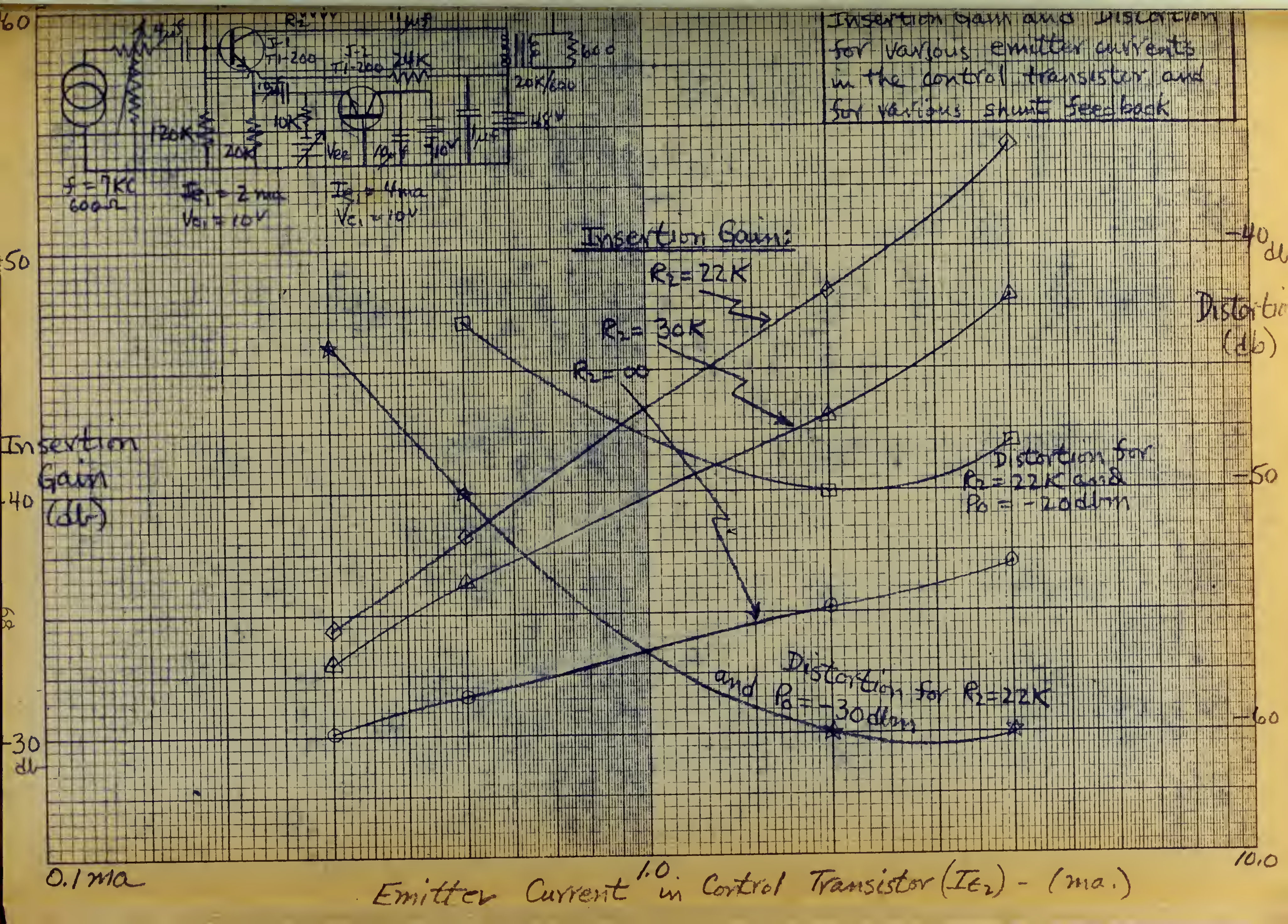
cases; in the last case, due to the low input impedance of the circuit, a very large capacitor must be employed to present an effective short circuit.

With the ratios obtained here about 18 db change of gain can be expected with no positive feedback. Unfortunately, for the very high values of resistance in the first and last cases, the negative series feedback is so great that, even with the shunt feedback resistance equal to zero, the positive feedback has no effect on the slope of the control characteristic. In addition to this, the insertion loss of the circuit is quite high. In actual tests of a circuit employing the output resistance of a common emitter circuit as a variable resistance, gain variations up to 20 db with distortion below 40 db have been obtained. (Figure 26) These tests were conducted at audio frequencies primarily due to the difficulty of obtaining suitable chokes at higher frequencies, the chokes being required by the method of capacitative coupling used. The possibility of using transformer coupling should be considered, but this is an extremely difficult design problem, the requirements of high secondary impedance, the wide range of terminating impedance, and high frequency response being difficult to achieve in a single transformer. A further disadvantage of this circuit is the fact that the output impedance of a common emitter circuit changes not only in magnitude but also in phase at frequencies as low as 0.01 $f_{\alpha o}$.


Insertion Gain and Distortion for various emitter currents in the control transistor
Insertion Gain (db)
-20
30
40
Distortion (db)
-30
-40
-50
-60
0.1 ma
1.0
10.0
Emitter Current in Control Transistor (I_{E_2}) - (ma)
Distortion for Po = 20 dbm
Distortion for Po = 30 dbm
Insertion Gain
f = 7 KC
600 Ω
120K
T-1
TI 200
T-2
TI 200
24K
20K/600
600
16V
20K
22.5V
48V

This has the effect of reducing the feedback, and therefore the control, with frequency and further distorts the frequency response of the amplifier.

Employing the very low impedances obtained from the input impedance of a common base circuit, we can apply positive feedback to increase the slope of the control characteristic and in addition we can see that we still have quite high gain in the circuit. Employing capacitative coupling between the control and the line amplifier transistors, low frequency gain variations of up to 20 db have been obtained with distortion less than -43 db. (Figure 27) Unfortunately the output power level to achieve this low distortion must be so low (about -30 dbm) that the input power levels to the stage are very low, in the region of noise power levels. Raising the power levels 10 db decreases the range of control to achieve the same distortion by about 4 db; these levels are still too low to be satisfactory and further increases lead to decreased control for a given amount of distortion. Here again transformer coupling can be used between the two transistors; although the design problem is still difficult, transformers having the desired characteristics can be obtained. The greatest advantage of employing a transformer here is that the variable input resistance of the control transistor can be stepped up to values where the stage gain is not so high and more reasonable power levels can be used. It might be argued that the stage gain could


Insertion Gain and Distortion for various emitter currents in the control transistor and for various shunt feedback
Insertion Gain:
R2 = 22K
R2 = 30K
R2 = ∞
Distortion for R2 = 22K and Po = -20dbm
Distortion for R2 = 22K and Po = -30dbm
Insertion Gain (db)
Distortion (db)
f = 7KC
600Ω
Ie1 = 2 ma
Vc1 = 10V
Ie1 = 4ma
Vc1 = 10V
T-1 TI-200
T-2 TI-200
24K
20K/600
600
120K
20K
10K
Vee
48V
60
50
40
30
db
-40 db
-50
-60
0.1 ma
1.0
10.0
Emitter Current in Control Transistor (Ie2) - (ma.)

be reduced to this point without the use of an impedance-step-up transformer by the use of negative shunt feedback around the transistor. This is true, but unfortunately such feedback sharply reduces the slope of the gain-series feedback resistance curve and a given resistance variation produces only small gain changes. Again in this method the effect on magnitude and phase of the input impedance of the common base transistor of increasing frequency must be considered for broad band amplifier applications, but fortunately they become appreciable only at frequencies greater than about 0.05 $f_{\alpha c o}$.

The power required to vary the emitter current of a transistor between 0.5 ma. and 4.0 ma. can be made as low as 4 mw. or less, thus offering some advantages over the use of a thermistor, but the variations of input impedance of transistors in magnitude and phase with frequency limit the use of transistors for control resistances to quite low frequencies. Another disadvantage is the fact that distortion of the signal may occur when the input signal is at a high level. This occurs, in spite of the fact that the emitter current of the line amplifier is at a high enough value to minimize distortion, because for high level signals the feedback resistance required is greatest, thereby requiring the smallest emitter current in the control transistor. The a-c signal applied to this transistor may be large enough compared to the low emitter current

to cause serious distortion.

Another method of varying gain is to vary the negative shunt feedback around an amplifier. The formulae for input resistance, current amplification and power gain for both the common emitter and common base circuits are derived in Appendix II and the derivation will not be repeated here. Considering the common emitter circuit when $r_c \gg R_s$, and $R_1 \approx R_s$,

$$G \approx \frac{a n r_c R_s}{\left[n^2 R_l r_b + r_c\{r_e + r_b(1-a)\}\left\{1 + \frac{R_l}{R_s}\right\}\right]} \qquad (6\text{-}13)$$

and the gain versus shunt feedback resistance curve will have a -3 db/octave slope. And further as R_s becomes small compared to R_1,

$$G \approx \frac{a n R_s^2}{R_l\left[r_e + r_b(1-a)\right]} \qquad (6\text{-}14)$$

and the slope of the curve will approach -6 db/octave. For the condition where $R_1 \approx R_s$,

$$r_i \approx \frac{\left\{n^2 R_l r_b + r_c\left[r_e + r_b(1-a)\right]\left[1 + \frac{R_l}{R_s}\right]\right\} R_s}{n a r_c R_l} \qquad (6\text{-}15)$$

and the input resistance is approximately linear with R_s, although of a fairly small value. For values of source impedance much greater than r_i, the loss due to mismatch

will also be linear with r_i, contributing a -3 db/octave slope to the insertion gain versus shunt feedback resistance curve. When R_s becomes much less than R_1, r_i approaches a constant value of very small magnitude,

$$r_i \approx \frac{r_e + r_b(1-a)}{na} \qquad (6\text{-}16)$$

and there is no further increase in the mismatch loss.

From the above theory we would expect to find that the insertion gain-shunt feedback resistance curve would have about a -6 db/octave slope for values of R_s in the order of R_1 and less. This theoretical variation is borne out experimentally, as shown in Figure 28.

When the common base circuit is considered, however, the results are somewhat different. Equation (6-13) is not obtained until $r_c \gg R_s$ and $R_1 > R_s$, although when $R_1 \gg R_s$, equation (6-14) is obtained. Thus the transition from a small slope to a -6 db/octave slope is much more rapid than that occuring in the common emitter circuit and also occurs at lower values of R_s. In similar fashion equation (6-15) for the input resistance is not obtained until $R_1 > R_s$, although again when $R_1 \gg R_s$, equation (6-16) is obtained. Thus the contribution of the mismatch loss is also to make the transition of slopes more rapid and to occur at lower values of R_s. These results were verified experimentally as seen from Figure 29. This

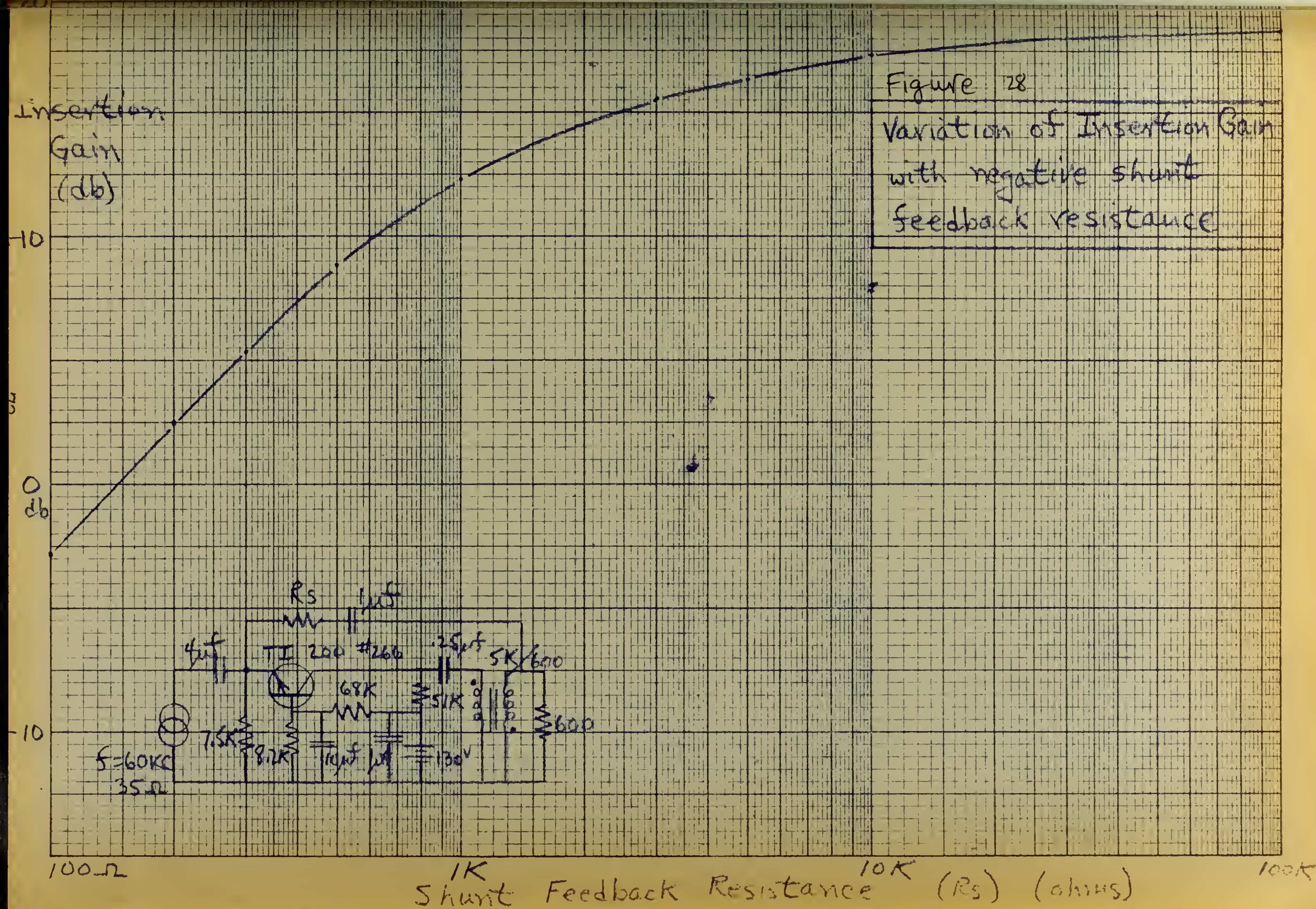
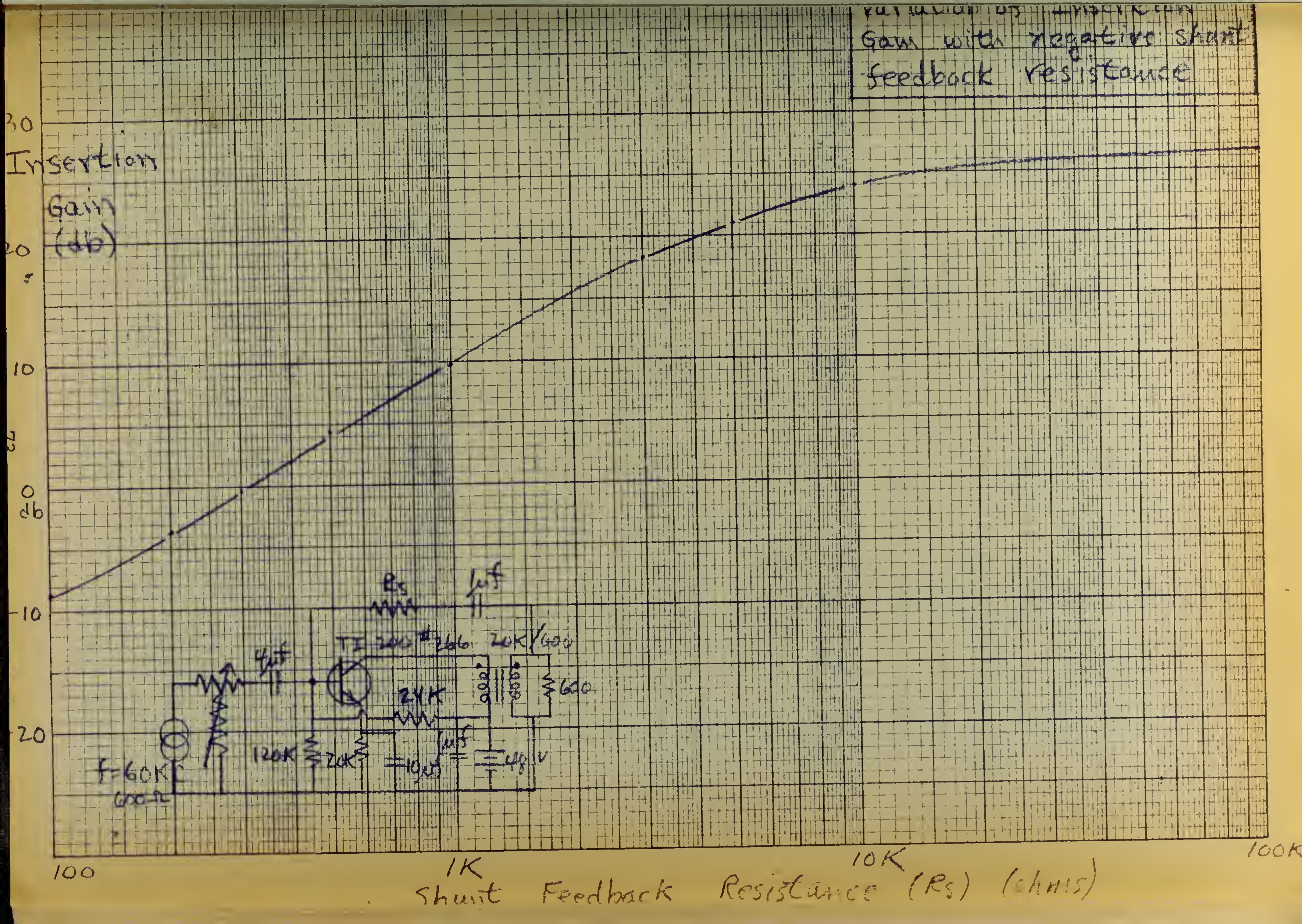


Figure 28

Variation of Insertion Gain with negative shunt feedback resistance


Variation of Insertion
Gain with negative shunt
feedback resistance
30
Insertion
Gain
(db)
20
10
0
db
-10
-20
100
1K
10K
100K
Shunt Feedback Resistance (Rs) (ohms)
Rs
1μf
TI
2N
#266
20K/600
4μf
24K
600
120K
20K
10μf
1μf
f=60K
600Ω

circuit does not offer as much attraction as the common emitter circuit, however, for the range of maximum slope does not occur in the region of variation that a thermistor would produce, and does not occur at such high insertion gains as does that of the common emitter circuit. Since the feedback is comparatively heavy, the frequency response, the distortion, and the variations of gain due to substitution of transistors of different alphas should be small in both circuits.

Thus, using a thermistor as the shunt feedback resistance in a common emitter circuit, a change of gain of 22 db can be obtained by a thermistor current variation of 0.5 ma. to 10.0 ma., with acceptable distortion and frequency response. The power requirement of 20 mw. is high but merely necessitates a d-c amplifier in the control loop.

The two best methods of changing gain by controlling feedback resistances appear then to be those employing a thermistor as either a shunt feedback resistance or a series feedback resistance. The change of gain obtainable is about the same for the two methods, although at low frequencies or at a single frequency the latter method is capable of much higher gain variations through the expedient of adding positive shunt feedback. The frequency response of the former method appears to be slightly better than that obtained when series feedback resistance variation is employed. The former method requires a high source impedance

while the latter requires a low source impedance.

There is no reason why either type of control could not be cascaded to obtain a greater range of control, provided that the thermistors employed, which must economically be in series for direct current, are sufficiently isolated to alternating currents. Chokes used for this purpose must be large enough so that frequency response is not impaired through their use. It might also be noted that when using series feedback controlled stages a small shunting resistance ($\approx$ 3K) must be used between stages and when using shunt feedback controlled stages a large series resistance ($\approx$ 2K) must be used between stages. These are for the purpose of insuring that input resistance variations of the second stage will not produce deleterious gain variations of the first stage due to changing its load resistance, and in both cases they result in lowering the insertion gain slightly.

CHAPTER VII

CONCLUSIONS

In summation it might be profitable to review the results obtainable by the various methods expounded in the preceding chapters. Of these, the use of tetrode transistors is not practical at this time, by virtue of their high cost and complete lack of replaceability.

Of the remaining methods, it can be seen that the practical range of control is about 20 db for either a variation of emitter current, of amplifier load resistance, or of a series or shunt feedback resistance. The only method which offers any advantage here is that in which the series feedback resistance is varied, since as has been proven, by the addition of positive shunt feedback the range of control can be extended to 30 db or 40 db at a single frequency or at audio frequencies.

Unfortunately this latter method results in additional distortion being introduced due to the positive feedback, although the distortion produced is not excessive except for the smallest values of series feedback resistance. Not so easily dismissed is the distortion which is inherent in the method employing a variable emitter current; this is the limiting feature of this method and prevents its use in applications having very low distortion requirements. It still remains probably the best method for use in com-

munications receivers by virtue of the facts that it requires fewer components and less control power. The distortion produced by the other methods is due only to too large a signal input and can be greatly minimized in the design of the systems.

The input signal level must be quite low for even moderate amounts of distortion when employing the method of varying emitter current, an unimportant fact as far as its use as a communications receiver is concerned but which is a further disadvantage for its use in other applications. For the other methods the signal level may be moderately high, since the systems act as amplifiers over part of their range and as attenuators over part.

The stiffness ratio of any of these systems is not directly dependent on the method of varying the gain of the transistor amplifier but rather on the method of error detection and amplification, a facet of the subject which has not been treated in this thesis. [4, 12] However, it should be noted that for systems employing thermistors not only is a d-c amplifier required in the control loop to furnish the necessary maximum current through the thermistors but also this amplifier is required to give stiffness to the system, since the thermistor current must vary by a factor of 10/1 or 20/1.

When temperature changes are considered, the variable emitter current method is again seen to be at a disadvantage

and would definitely require the use of silicon transistors in the amplifier. For the other methods, proper temperature stabilization, such as described by Sherr and Kwap, [10] will cause temperature changes to have a negligible effect on the operation of the system. The variations of thermistor resistance with temperature will cause the system to regulate at a different point but will cause only a negligible change of output level as long as the thermistor current is able to decrease as the temperature increases, in order to maintain a constant thermistor resistance. Once the thermistor current drops to zero the system will no longer regulate and control is lost; this practically sets a minimum limit to the thermistor current of about one ma. Of course, in any system employing an amplifier in the control loop, any variation in the gain of the amplifier due to temperature or any other cause will not be corrected by the system and will result in an error in the output level. Due to the drift of the output current of transistor d-c amplifiers, a drift believed to be due primarily to temperature effects, the use of a silicon transistor in the control loop amplifier will be mandatory.

It should be noted also that when the input level to the amplifier increases, a decrease of d-c emitter current is needed for the variable emitter current technique in order to decrease the amplifier gain. This is also true when using a variable series feedback resistance where the thermistor

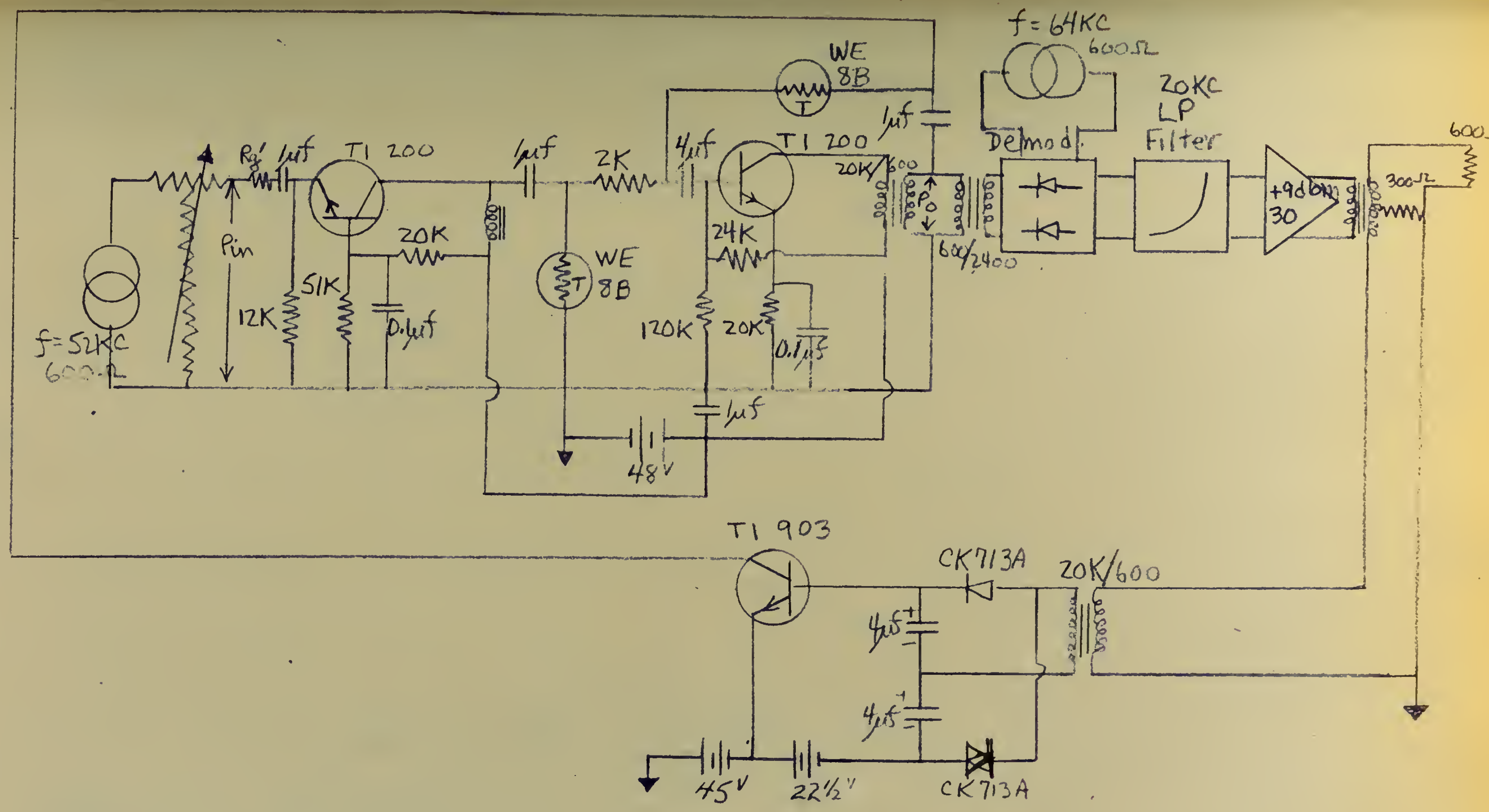
current must decrease in order to reduce the amplifier gain. When using a variable load resistance or a variable shunt feedback resistance, the opposite is true: the thermistor current must increase in order for the amplifier gain to decrease. A consideration of these facts is necessary when the use of a combination of these four methods is contemplated in order to increase the range of control of the system, since a design requiring that a control current increase in one section of the system and at the same time decrease in another section would be not only difficult but also complex.

There would seem to be no merit in combining the variable emitter current method and the method of variable series resistance into a single system, since this combination would offer no significant advantages and would combine the disadvantages and limitations of both systems. However, it is practical to combine into a single system the methods of variable load resistance and shunt feedback resistance and achieve certain advantages. One of these is the greater ease with which the d-c thermistor current may be fed through the system, since all chokes may be eliminated from the control circuit. Secondly, for practical values of load resistance, the input resistance of a common base stage is essentially constant, a particular advantage if transformer coupling is used for the input to the automatic gain control system. Thirdly, no interstage transformers are required in the system.

An automatic gain control system employing these latter two methods in combination is shown in Figure 30. Here one thermistor is employed as a variable load resistance for the first stage, a common base circuit. Another thermistor is used as a shunt feedback resistance around the second stage, a common emitter circuit. There is a two kilohm resistance between the two stages: this insures that the load resistance of the first stage is primarily due to the first thermistor and also insures that the source impedance seen by the input terminals of the second stage is much greater than the input resistance of that stage. Both these effects could be increased by the use of a larger interstage resistance, but the additional attenuation thus introduced more than offsets the slight increase in the range of control obtainable.

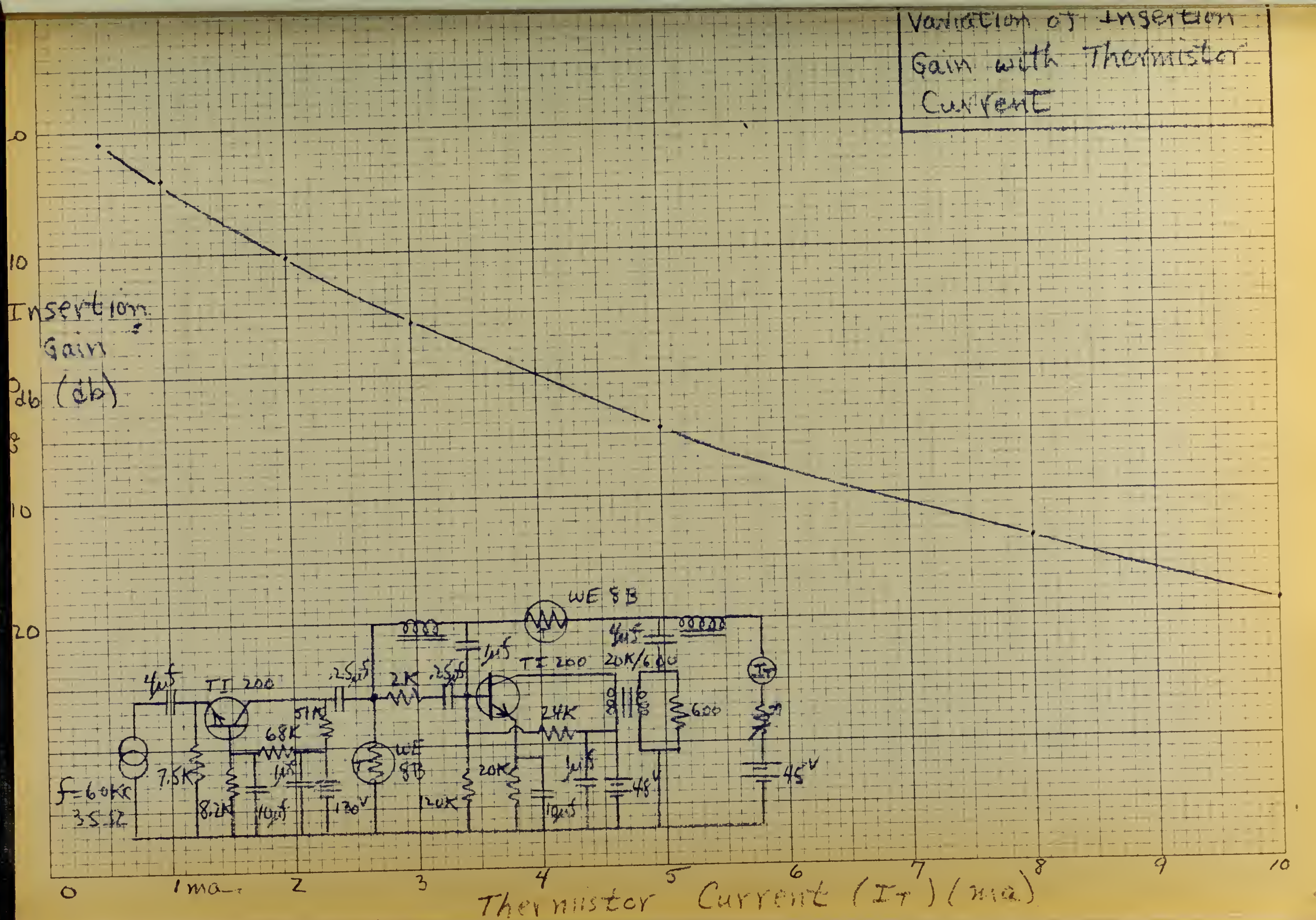
There are thus four mechanisms occuring in this circuit: a variable load resistance on the first stage, a variable power division between the first thermistor and the subsequent circuitry, a variable degree of mismatch at the input of the second stage due to the change of its input resistance, and a variable negative feedback around the second stage. From previous theory and experimental results we expect in the nature of a 40 db change of insertion gain of this system and open loop tests of the combination method verify this. (Figure 31)

In the closed loop system the output of the amplifier



Low Distortion, Broadband Transistor Amplifier
Automatic Gain Control System

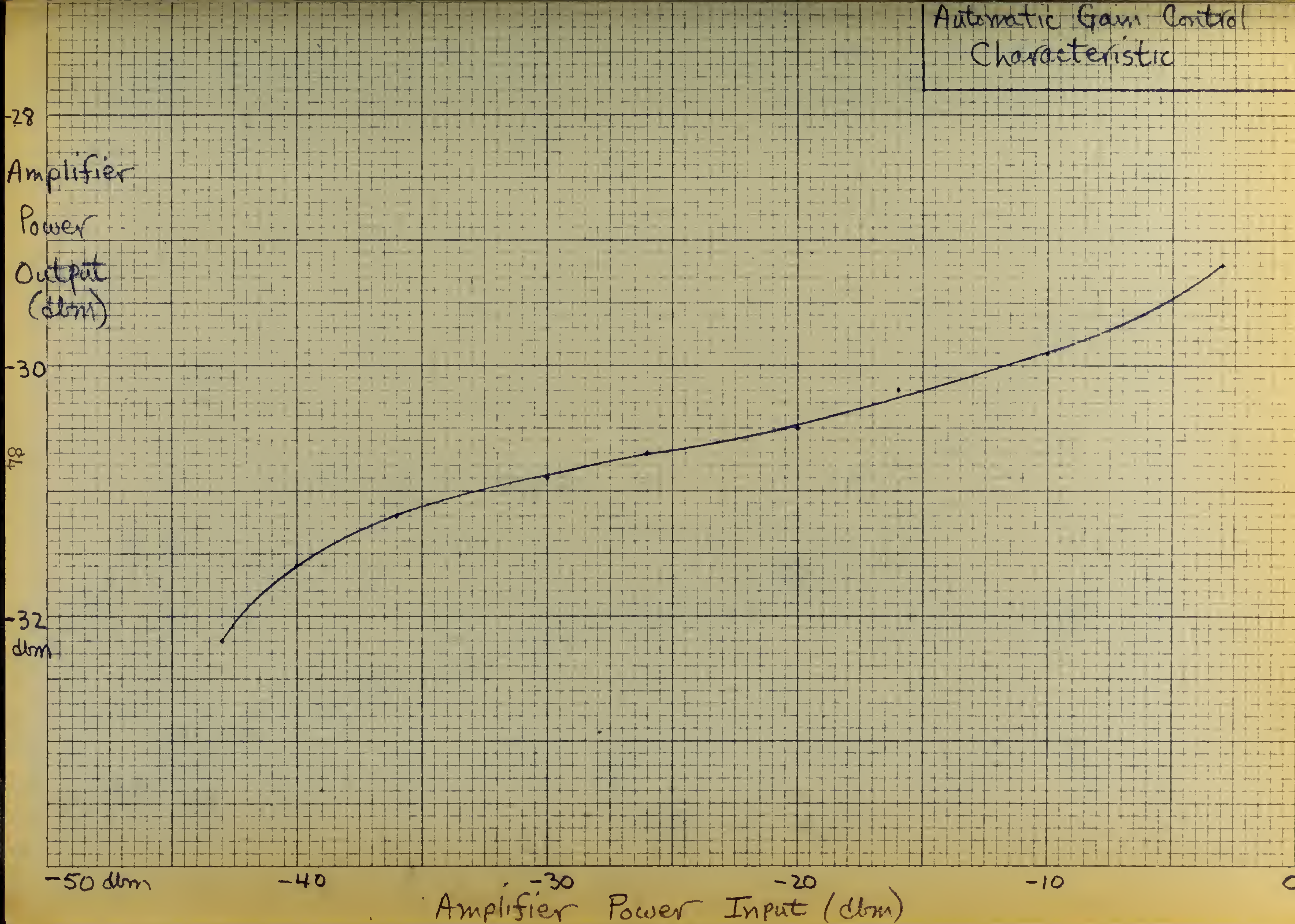
Figure 30

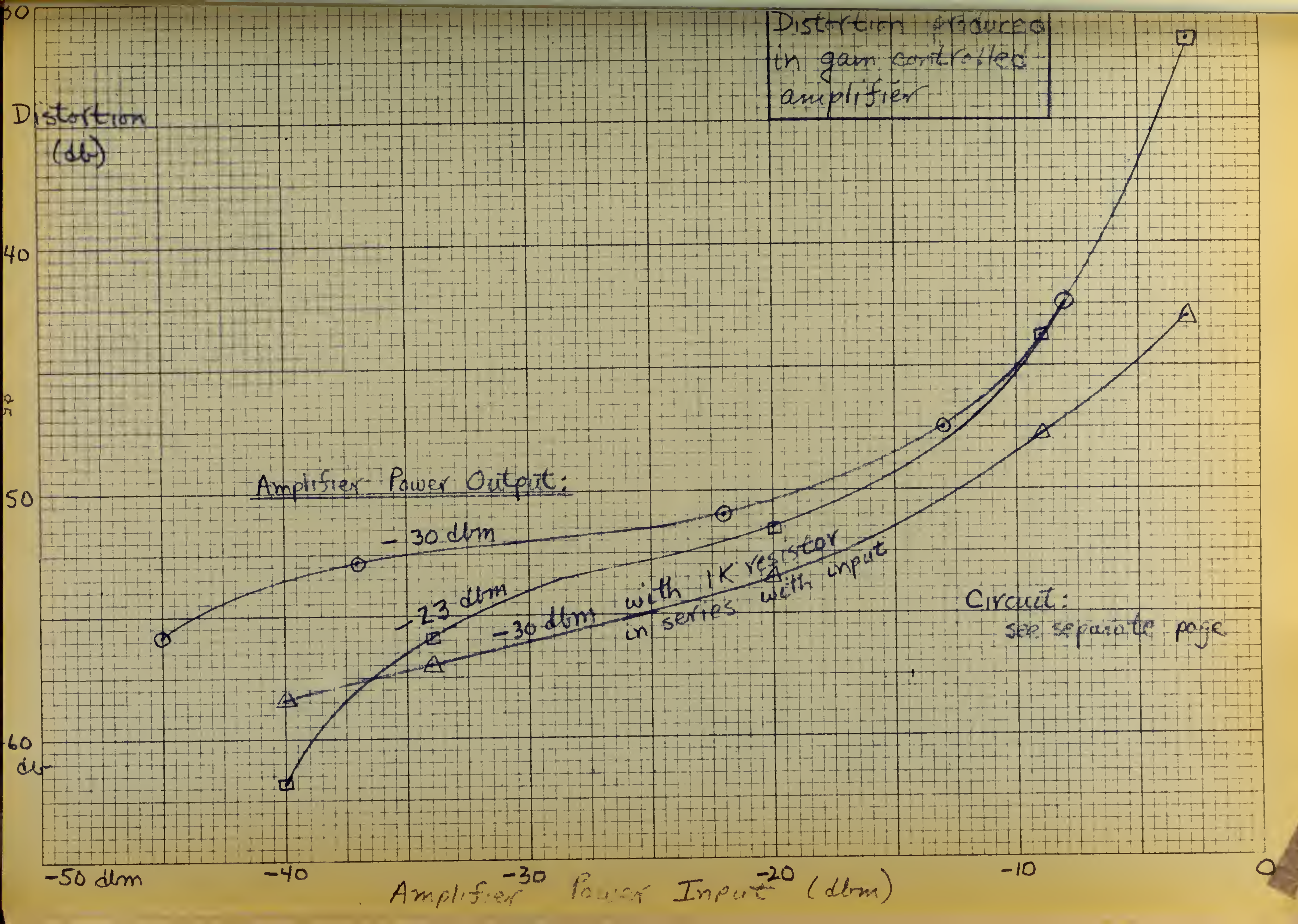

Variation of Insertion Gain with Thermistor Current
Insertion Gain (db)
0
10
20
0
1 ma
2
3
4
5
6
7
8
9
10
Thermistor Current (I_T) (ma)
f=60kc
35Ω
4μf
TI 200
7.5K
8.2K
68K
51K
10μf
130V
.25μf
2K
WE 8B
20K
1μf
24K
20K/600
600
48V
45V
I_T

is fed through a demodulator and another amplifier and from this point, 4 mw. of pilot power is subtracted and rectified in a voltage doubler. The rectified pilot is used, through a back-biasing arrangement, to control the input signal to a d-c amplifier, here a silicon transistor. This amplifier supplies the d-c current to the thermistors.

As can be seen from Figure 32, for the system described, a 40 db change in input level produced only a 3 db change of output level, a stiffness ratio of 13.3. In the mid-region of the automatic gain control characteristic the stiffness ratio was twenty. The distortion curves for this circuit show that for high power inputs the distortion begins to become appricialbe for the series input resistance R_g' equal to zero. (Figure 33) The addition of a small amount of resistance here ($R_g' = 1K$) greatly reduces the distortion produced, as can be seen from the figure, although also reducing the system insertion gain. This is the distortion curve corresponding to the automatic gain control characteristic shown in Figure 32, and it can be seen that the maximum distortion produced in the automatic gain control system is -43 db, and over most of the range of input level variation is less than -50 db. The effect of heating such a system showed that for temperatures up to 60° C there was only a 0.9 db change of output level.

In conclusion then, of the systems discussed in this thesis, the most commercially practical appear to be the


Automatic Gain Control
Characteristic
-28
Amplifier
Power
Output
(dbm)
-30
-32
dbm
-50 dbm
-40
-30
-20
-10
0
Amplifier Power Input (dbm)



Distortion produced in gain controlled amplifier
Distortion (db)
30
40
45
50
60 db
Amplifier Power Output:
-30 dbm
-23 dbm
-30 dbm with 1K resistor in series with input
Circuit: see separate page
-50 dbm
-40
-30
-20
-10
0
Amplifier Power Input (dbm)

cascaded arrangement of stages employing a variable d-c emitter current, where distortion and temperature effects are negligible, and the combination described in the preceding paragraph where the requirements for low distortion and very small changes of control with temperature changes are very stringent.

Undoubtedly the methods herein outlined can be extended and improved, but they serve as a starting point in the practical design of an automatic gain control system for a transistor amplifier and prove that it is possible to achieve automatic gain control of a transistor amplifier to a degree capable of meeting typical design specifications and with a proficiency capable of competing with vacuum tube systems.

BIBLIOGRAPHY

1.	Barton, Loy E.	An Experimental Transistor Personal Broadcast Receiver, Proceedings of the IRE, Vol. 42, No. 7, July, 1954, pp. 1062-1066.
2.	Bode, H. W.	Network Analysis and Feedback Amplifier Design, D. Van Nostrand Co., Inc., New York, 1945.
3.	-----	The Transistor: Selected Reference Material on Characteristics and Applications, Bell Telephone Laboratories, Inc., New York, 1951.
4.	Fett, Gilbert H.	Feedback Control Systems, Prentiss Hall, 1954.
5.	Krull, Alan R.	Transistors and Their Applications; A Bibliography, 1948-1953, Transactions of the IRE, Professional Group on Electron Devices, Vol. ED-1, No. 3, August, 1954.
6.	McKenzie, Alexander A.	"Electrons at Work" (feature), Electronics, Vol. 26, No. 8, August, 1953, pp. 202-205. (Abstract from "Applications of Transistors to Radio Receiver Circuitry" by Emerick Toth at Colloquium on Transistors in Theory and Practice, Naval Research Laboratory, Washington, D. C.)
7.	Oakes, J. B.	Some Transistor Circuit Applications, Notes on Transistor Physics and Electronics, Johns Hopkins University Applied Physics Laboratory, 1953, pp. 255-262.
8.	Shea, Richard F.	Principles of Transistor Circuits, John Wiley & Sons, New York, 1953.
9.	Shea, Richard F.	Transistor Audio Amplifiers, John Wiley & Sons, New York, 1955.
10.	Sherr, Sol and Kwap, Theodore	Stabilizing Transistors, Tele-Tech and Electronic Industries, Vol. 14, No. 3, March, 1955, pp. 74 ff.

11. Shockley, W. — Electrons and Holes in Semi-Conductors, D. Van Nostrand Co., Inc., New York, 1950.

12. Smith, D. H. — The Suitability of the Silicon Alloy Junction Diode as a Reference Standard in Regulated Metallic Rectifier Circuits, AIEE Transactions, Part I, Communications and Electronics Section, No. 16, Jan., 1955, pp. 645-651.

13. ----- — "As We Go To Press" (feature), Editorial Staff, Tele-Tech and Electronic Industries, Vol. 13, No. 12, Dec., 1954.

14. Wallace, R. L., Schimpf, L. G., and Dickten, E. — High Frequency Transistor Tetrode, Electronics, Vol. 26, No. 1, Jan., 1953, pp. 112-113.

15. Webster, W. M. — On the Variation of Junction-Transistor Current-Amplification Factor with Emitter Current, Proceedings of the IRE, Vol. 42, No. 6, June, 1954, pp. 914-920.

16. Young, Richard L. and Steffens, Ralph — Guide to Transistor Literature, 1954, Glenn L. Martin Company, Baltimore, 1954.

APPENDIX I

VARIATIONS OF TRANSISTOR EQUIVALENT CIRCUIT PARAMETERS WITH EMITTER CURRENT AND COLLECTOR VOLTAGE

Equipment employed:	Lenkurt Test Set 15 Transistor Hybrid Parameter Test Set
	Hewlett Packard 650A Audio Oscillator
	Hewlett Packard 200C VTVM
	Hewlett Packard 300A Harmonic Analyzer
Employment of transistors:	TI 200 #265: Tests A,B,C,D TI 200 #266: Tests E,F,G,H TI 903 #3 : Tests J,K,L,M
Test operating conditions:	Emitter currents and collector voltages were identical during: Tests A,E,J; Tests B,F,K; Tests C,G,L; and Tests D,H,M.

Formulae for calculation of equivalent circuit parameters:

For the common base circuit:

$$r_e = h_{11} - \frac{h_{12}}{h_{22}}\left(1 - h_{21}\right)$$

$$r_b = \frac{h_{12}}{h_{22}}$$

$$\alpha = \frac{h_{21} - h_{12}}{1 - h_{12}} \approx h_{21}$$

$$r_c = \frac{1 - h_{12}}{h_{22}} \approx \frac{1}{h_{22}}$$

APPENDIX I

Ie (ma)	Vc (v)	h_{11} (ohms)	h_{12} x10^{-5}	h_{21} ---	h_{22} (umhos)	i_{co} (ua)	r_e (ohms)	r_b (ohms)	r_c (ohms)	alpha ---
			TEST A		TI 200 #265					
2.0	10.0	20.6	7.0	.955	1.13	5	17.8	62.0	885 K	.955
1.5	12.2	27.4	7.0	.955	1.12	5.5	24.6	62.5	893 K	.955
1.2	13.2	31.0	6.5	.952	1.10	5.5	28.2	59.1	910 K	.952
0.8	15.1	43.5	6.0	.948	1.10	6	40.7	54.6	910 K	.948
0.5	16.4	64.5	5.5	.940	1.08	6.5	61.5	50.4	917 K	.940
0.2	17.6	140.	5.0	.930	1.08	7	136.5	50.4	917 K	.930
0.1	18.0	276.	5.0	.910	1.08	7	271.5	50.4	917 K	.910
			TEST B		TI 200 #265					
1.0	5.0	34.5	8.0	.941	1.07	4.5	30.1	74.1	925 K	.941
0.7	5.8	47.0	7.5	.938	1.07	5	42.7	69.5	925 K	.938
0.5	6.4	64.0	7.5	.935	1.07	5	59.5	69.5	925 K	.935
0.3	6.9	101.	7.0	.930	1.07	5.5	98.5	64.8	925 K	.930
0.2	7.2	142.	7.0	.925	1.07	5.5	136.	64.8	925 K	.925
0.1	7.3	275.	7.0	.910	1.07	5.5	269.	64.8	925 K	.910
0.07	7.45	392.	7.0	.900	1.07	5.5	386.	64.8	925 K	.900
0.04	7.5	565.	7.0	.895	1.07	5.5	559.	64.8	925 K	.895
			TEST C		TI 200 #265					
0.3	5.5	100.	8.0	.930	1.07	5	94.8	74.1	925 K	.930
0.175	5.75	194.	8.0	.920	1.07	5	188.	74.1	925 K	.920
0.125	5.90	218.	8.0	.915	1.07	5	211.	74.1	925 K	.915
0.10	5.95	272.	8.0	.912	1.07	5	265.	74.1	925 K	.912
0.05	6.0	502.	8.0	.900	1.07	5	495.	74.1	925 K	.900
0.03	6.1	570.	8.0	.895	1.07	5	562.	74.1	925 K	.895
			TEST D		TI 200 #265					
4.0	10.0	8.7	12.5	.985	1.32	5	7.4	87.1	758 K	.985
3.5	10.0	10.7	10.0	.980	1.28	5	9.1	78.2	781 K	.980
3.0	10.0	14.0	9.5	.978	1.22	5	12.4	73.9	820 K	.978
2.5	10.0	17.0	8.0	.975	1.18	5	15.3	67.8	847 K	.975
2.0	10.0	20.8	7.0	.957	1.12	5	18.1	62.5	893 K	.957
1.5	10.0	27.0	6.5	.955	1.10	5	24.3	59.1	910 K	.955
1.0	10.0	36.0	6.5	.950	1.08	5	33.2	55.1	917 K	.950
0.5	10.0	68.0	6.5	.940	1.07	5	64.7	55.1	925 K	.940

APPENDIX I

TEST E TI 200 #266

Ie (ma)	Vc (v)	h_{11} (ohms)	h_{12} $\times 10^{-5}$	h_{21}	h_{22} (umhos)	i_{co} (ua)	r_e (ohms)	r_b (ohms)	r_c (ohms)	alpha
2.0	10.0	21.5	8.0	.948	1.78	14	19.2	45.0	562 K	.948
1.5	12.2	26.1	7.5	.945	1.71	14.5	23.7	43.9	585 K	.945
1.2	13.2	30.0	7.5	.945	1.72	15	27.7	43.7	581 K	.945
0.8	15.1	41.0	8.0	.940	1.82	16	38.3	44.0	550 K	.940
0.5	16.4	62.5	8.0	.932	1.8[illegible]	17.5	59.6	42.6	532 K	.932
0.2	17.6	138.	8.0	.919	2.05	19	135.	39.0	488 K	.919
0.1	18.0	276.	8.0	.905	2.05	19	271.	39.0	488 K	.905

TEST F TI 200 #266

Ie (ma)	Vc (v)	h_{11} (ohms)	h_{12} $\times 10^{-5}$	h_{21}	h_{22} (umhos)	i_{co} (ua)	r_e (ohms)	r_b (ohms)	r_c (ohms)	alpha
1.0	5.0	34.0	7.0	.942	1.33	7	30.9	53.0	757 K	.942
0.7	5.8	45.5	8.0	.940	1.32	8	41.9	60.6	757 K	.940
0.5	6.4	62.5	7.5	.939	1.32	8.5	59.0	56.8	757 K	.939
0.3	6.9	100.	8.0	.930	1.32	9	95.8	60.6	757 K	.930
0.2	7.2	162.	8.5	.925	1.32	9	157.2	64.4	757 K	.925
0.1	7.3	278.	9.5	.920	1.32	9	272.	72.0	757 K	.920
0.07	7.45	395.	10.5	.910	1.32	9	388.	79.6	757 K	.910
0.04	7.5	610.	12.0	.905	1.32	9.5	601.	90.0	757 K	.905

TEST G TI 200 #266

Ie (ma)	Vc (v)	h_{11} (ohms)	h_{12} $\times 10^{-5}$	h_{21}	h_{22} (umhos)	i_{co} (ua)	r_e (ohms)	r_b (ohms)	r_c (ohms)	alpha
0.3	5.5	98.0	8.0	.930	1.29	7.5	93.7	62.1	775 K	.930
0.175	5.75	156.	9.0	.925	1.28	7.5	151.	69.9	775 K	.925
0.125	5.90	212.	9.0	.920	1.29	8	206.	69.9	775 K	.920
0.10	5.95	268.	10.0	.920	1.28	7.5	261.	77.8	775 K	.920
0.05	6.0	510.	11.0	.907	1.28	8.0	502.	85.5	775 K	.907
0.03	6.1	610.	12.0	.900	1.29	8	601.	93.2	775 K	.900

TEST H TI 200 #266

Ie (ma)	Vc (v)	h_{11} (ohms)	h_{12} $\times 10^{-5}$	h_{21}	h_{22} (umhos)	i_{co} (ua)	r_e (ohms)	r_b (ohms)	r_c (ohms)	alpha
4.0	10.0	13.8	10.0	.948	2.23	12	11.5	45.0	448 K	.948
3.5	10.0	15.0	10.0	.948	2.07	12	12.5	48.3	483 K	.948
3.0	10.0	16.4	9.0	.948	2.00	12	14.1	45.0	500 K	.948
2.5	10.0	18.3	8.5	.948	1.88	12	16.0	45.2	532 K	.948
2.0	10.0	21.3	8.0	.948	1.80	12	19.0	44.5	555 K	.948
1.5	10.0	26.0	8.0	.948	1.70	12	23.6	47.1	588 K	.948
1.0	10.0	33.0	8.0	.945	1.62	12	30.3	49.4	617 K	.945
0.5	10.0	62.5	8.0	.940	1.57	12	59.4	51.0	637 K	.940

APPENDIX I

Ie (ma)	Vc (v)	h_{11} (ohms)	h_{12} --- x10^{-5}	h_{21} ---	h_{22} (umhos)	i_{co} (ua)	r_e (ohms)	r_b (ohms)	r_c (ohms)	alpha ---
			TEST J	TI 903 #2						
2.0	10.0	23.0	4.0	.928	1.29	2	20.8	31.0	775 K	.928
1.5	12.2	30.5	4.0	.928	1.22	2	28.1	32.8	820 K	.928
1.2	13.2	38.0	4.0	.925	1.18	2.5	35.5	33.9	847 K	.925
0.8	15.1	60.0	4.5	.915	1.14	2.5	56.6	39.5	877 K	.915
0.5	16.4	100.0	5.0	.895	1.12	2.5	95.3	44.6	893 K	.895
0.2	17.6	300.	7.0	.815	1.11	2.5	288.	63.1	900 K	.815
0.1	18.0	625.	9.0	.725	1.10	2.5	603.	81.8	909 K	.725
			TEST K	TI 903 #2						
1.0	5.0	45.5	4.5	.920	1.20	2	42.5	37.5	833 K	.920
0.7	5.8	67.5	5.0	.908	1.17	2	63.6	42.8	855 K	.908
0.5	6.4	100.	6.5	.895	1.13	2	94.0	57.5	885 K	.895
0.3	6.9	198.	8.0	.860	1.11	2	188.	72.1	900 K	.860
0.2	7.2	350.	9.5	.825	1.10	2	335.	86.3	909 K	.825
0.1	7.3	625.	11.0	.735	1.09	2	598.	101.	917 K	.735
0.07	7.45	860.	11.0	.680	1.09	2	828.	101.	917 K	.680
0.04	7.5	1160.	11.0	.625	1.09	2	1122.	101.	917 K	.625
			TEST L	TI 903 #2						
0.3	5.5	194.	10.0	.865	1.09	2	182.	91.8	917 K	.865
0.175	5.75	415.	11.5	.810	1.09	2	395.	105.5	917 K	.810
0.125	5.90	470.	12.5	.770	1.09	2	444.	114.7	917 K	.770
0.10	5.95	610.	12.5	.730	1.08	2	579.	114.7	926 K	.730
0.05	6.0	1070.	12.5	.640	1.08	2	1029	114.7	926 K	.640
0.03	6.1	1140.	12.5	.625	1.08	2	1097	114.7	926 K	.625
			TEST M	TI 903 #2						
4.0	10.0	15.0	4.0	.925	1.59	2	13.1	25.2	629 K	.925
3.5	10.0	16.3	4.5	.930	1.48	2	14.2	30.4	676 K	.930
3.0	10.0	18.6	4.5	.930	1.40	2	16.4	32.1	715 K	.930
2.5	10.0	21.0	4.5	.930	1.32	2	18.6	34.1	757 K	.930
2.0	10.0	25.6	4.0	.930	1.26	2	23.4	31.7	794 K	.930
1.5	10.0	32.0	4.0	.928	1.20	2	29.6	33.4	833 K	.928
1.0	10.0	47.0	4.5	.920	1.17	2	43.9	38.5	855 K	.920
0.5	10.0	108.0	4.5	.895	1.13	2	103.8	39.8	885 K	.895

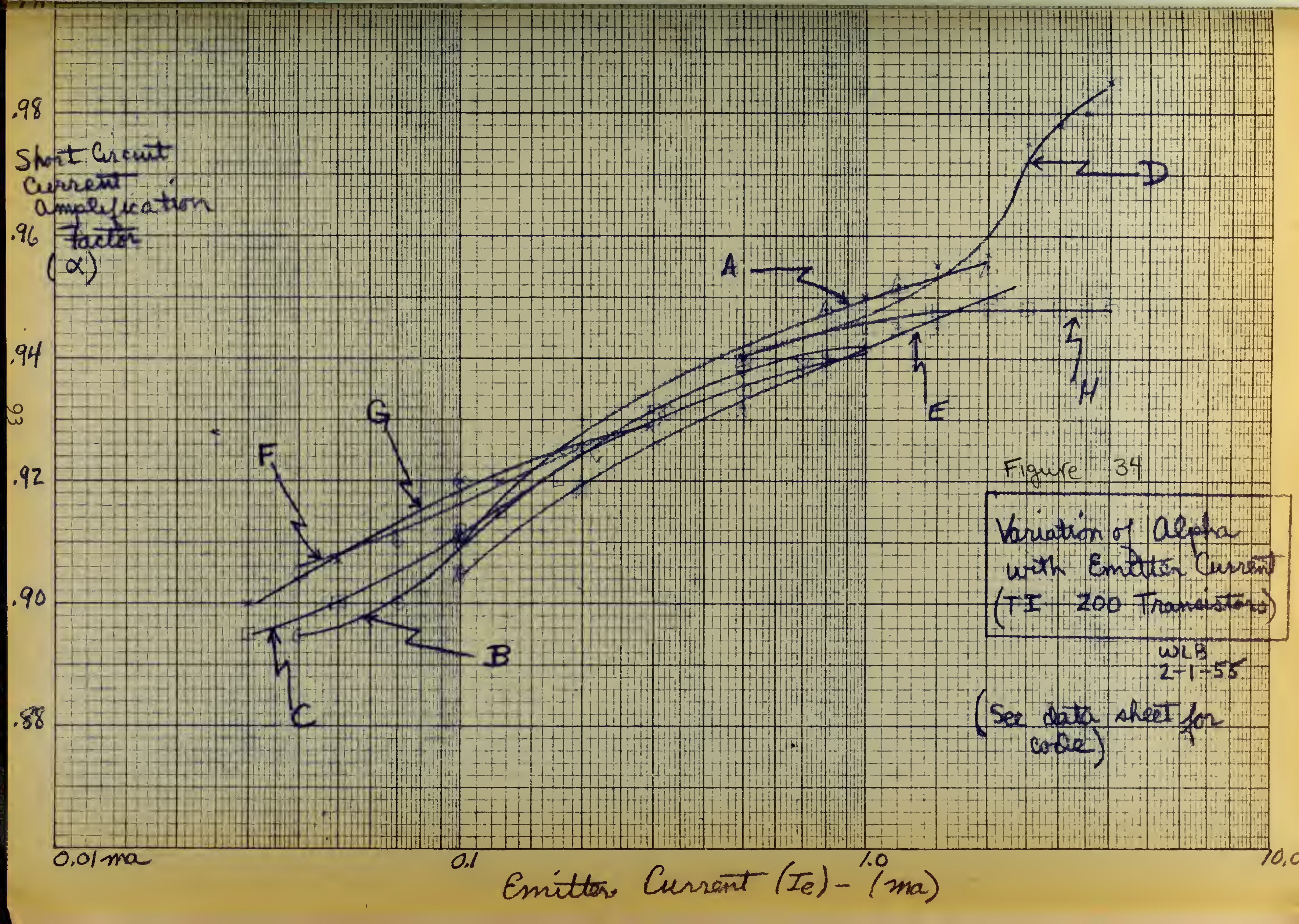
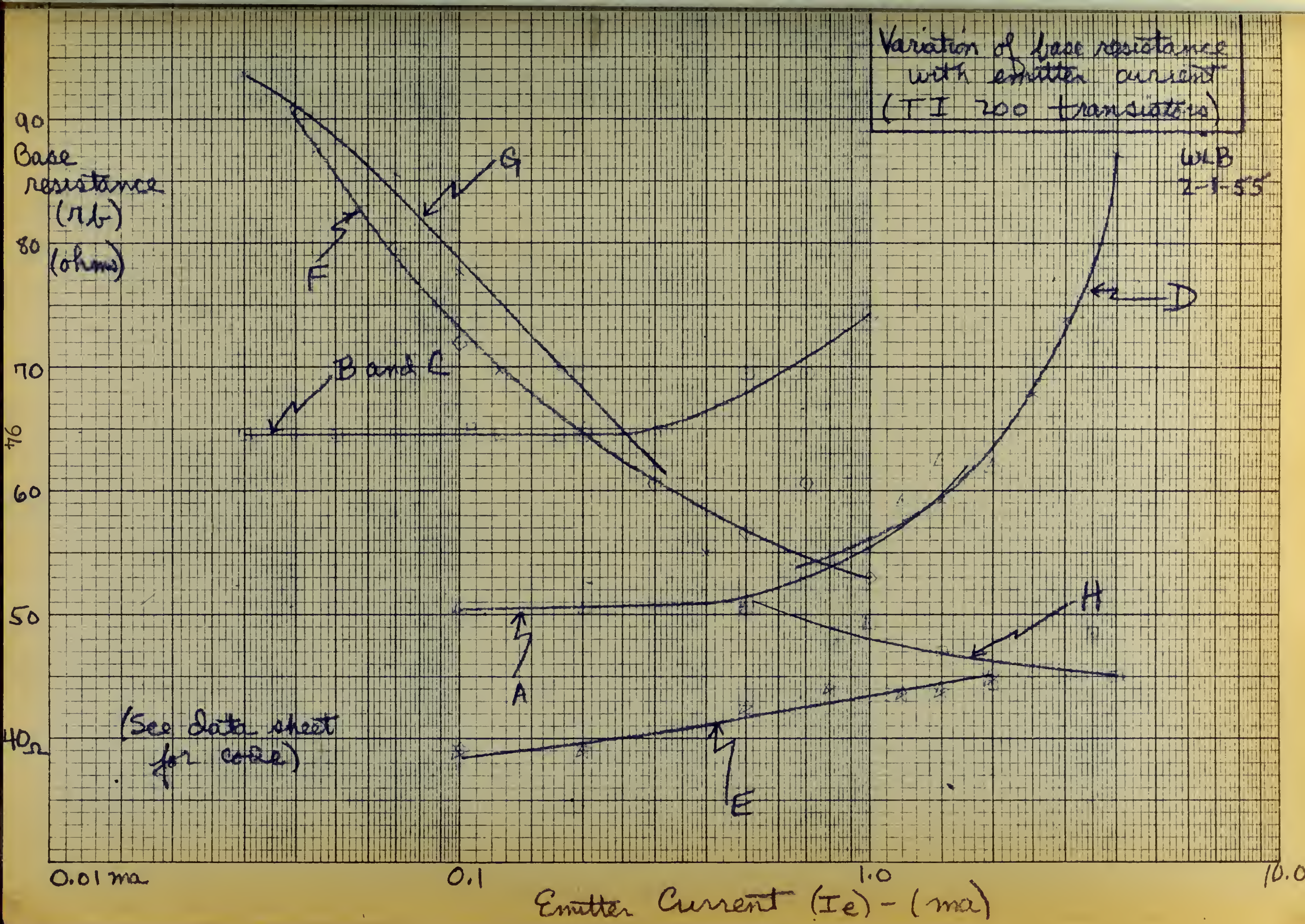


Figure 34

Variation of Alpha with Emitter Current (TI 200 Transistors)


Variation of base resistance
with emitter current
(TI 200 transistors)
WLB
2-1-55
90
Base
resistance
(rb)
(ohms)
80
70
60
50
40Ω
G
F
B and C
D
A
H
E
(See data sheet
for code)
0.01 ma
0.1
1.0
10.0
Emitter Current (Ie) - (ma)

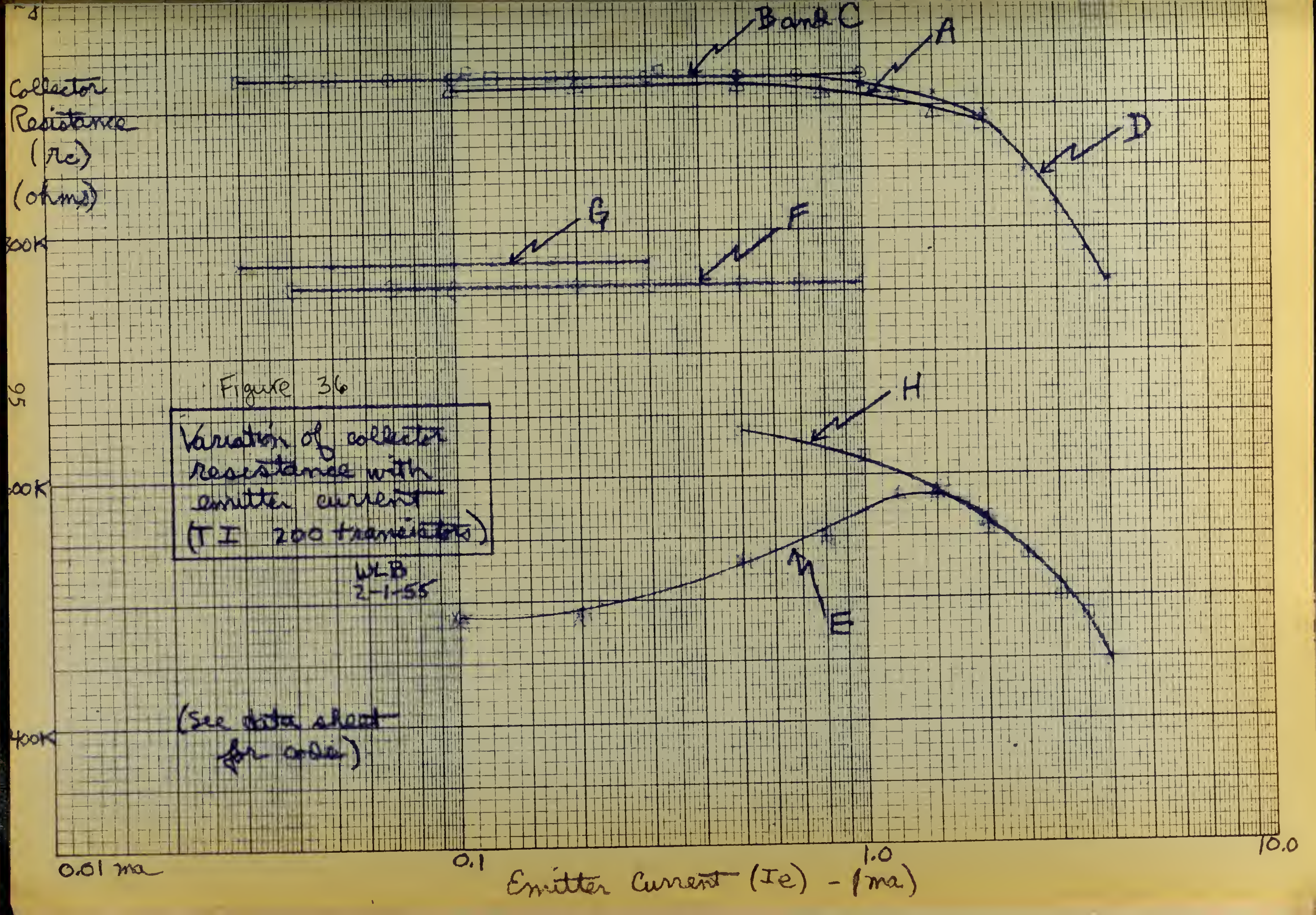


Figure 36

Variation of collector resistance with emitter current (TI 200 transistors)

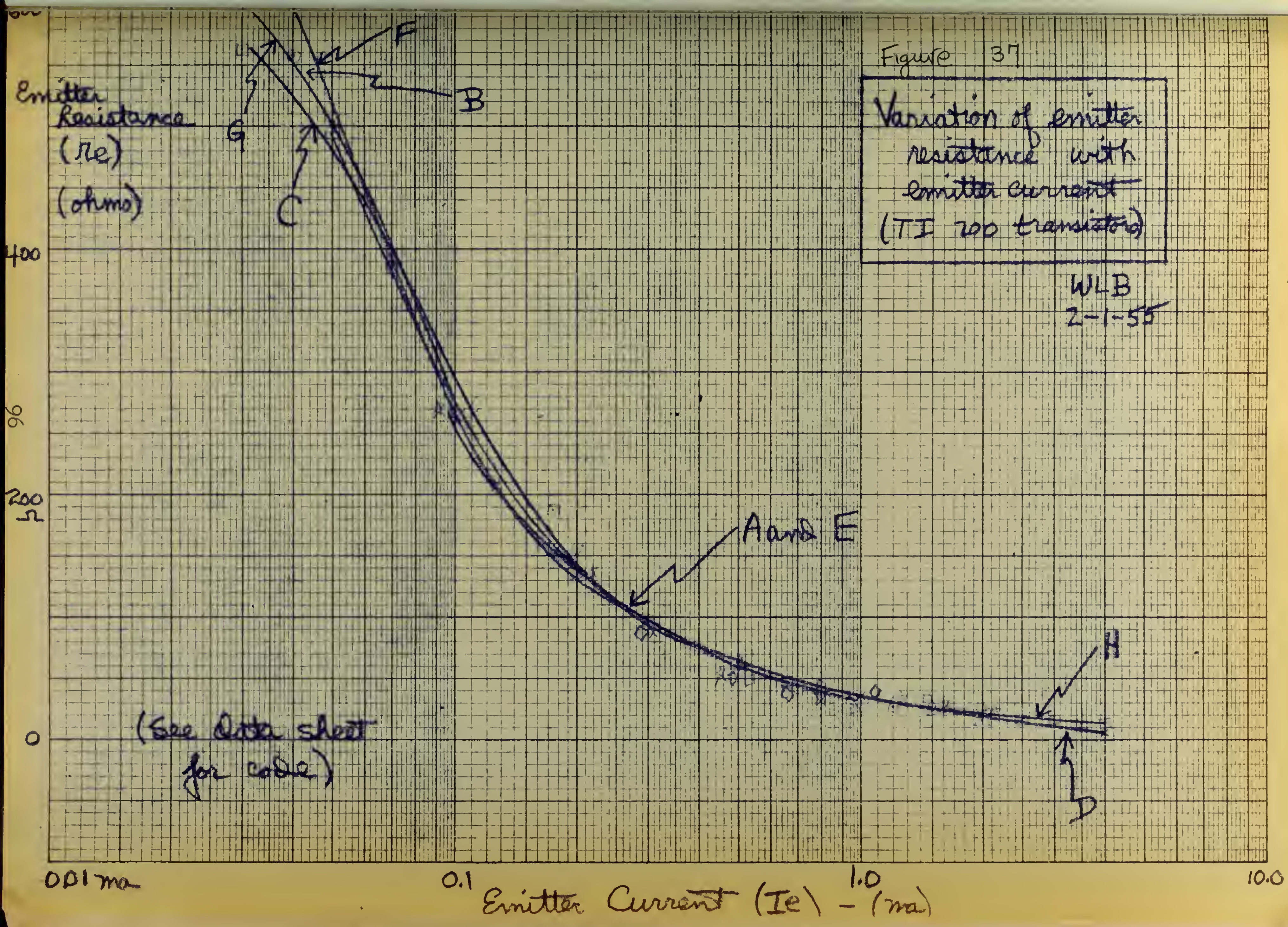
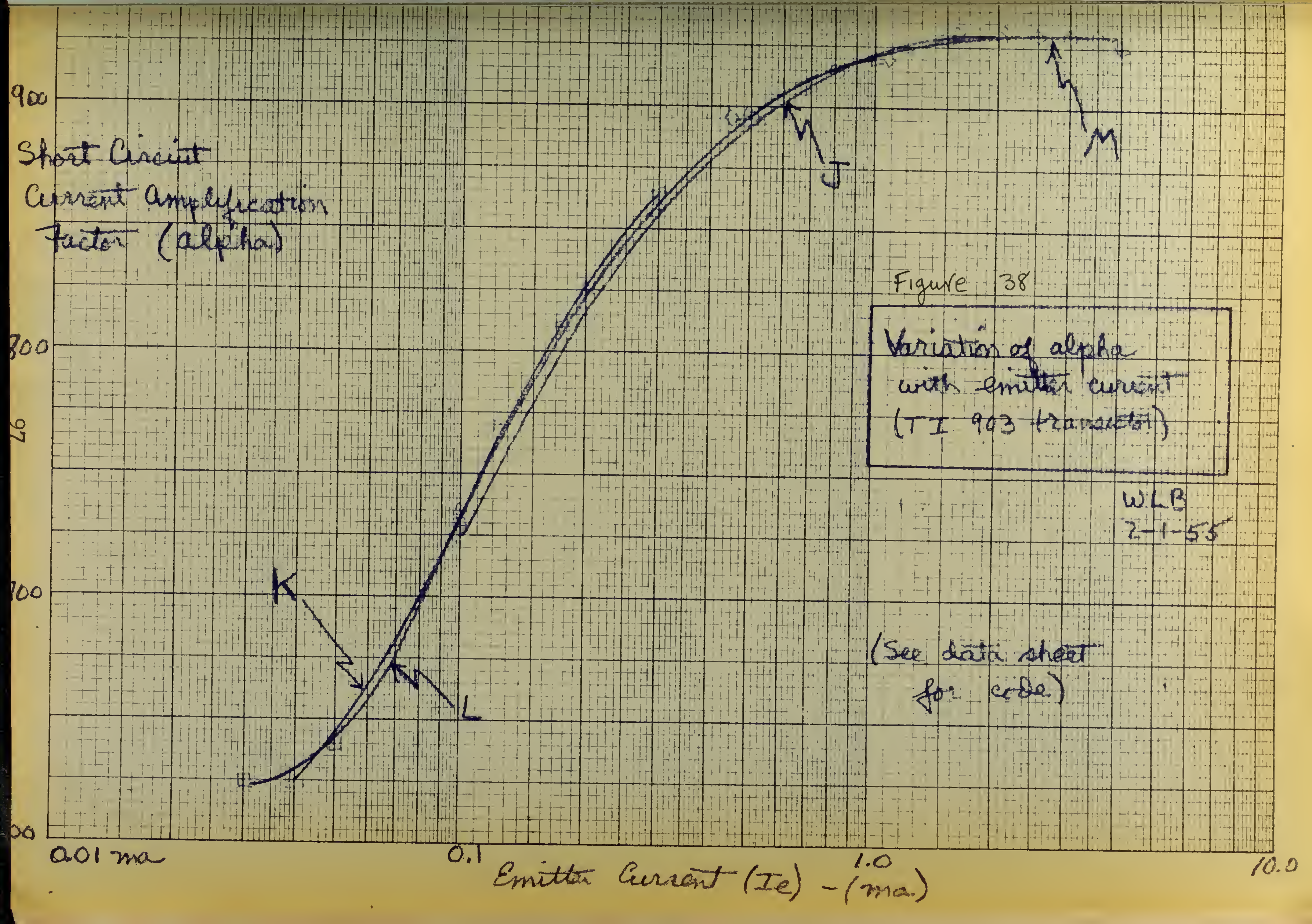


Figure 37
Variation of emitter resistance with ~~emitter current~~
(TI 200 transistors)
WLB
2-1-55
Emitter Resistance
(re)
(ohms)
400
200 Ω
0
0.01 ma
0.1
1.0
10.0
Emitter Current (Ie) - (ma)
A and E
B
C
D
F
G
H
(See data sheet for code)

96


900
Short Circuit
Current Amplification
Factor (alpha)
800
97
00
00
0.01 ma
0.1
1.0
10.0
Emitter Current (Ie) - (ma.)
K
L
J
M
Figure 38
Variation of alpha
with Emitter current
(TI 903 transistor)
W.L.B
2-1-55
(See data sheet
for code)

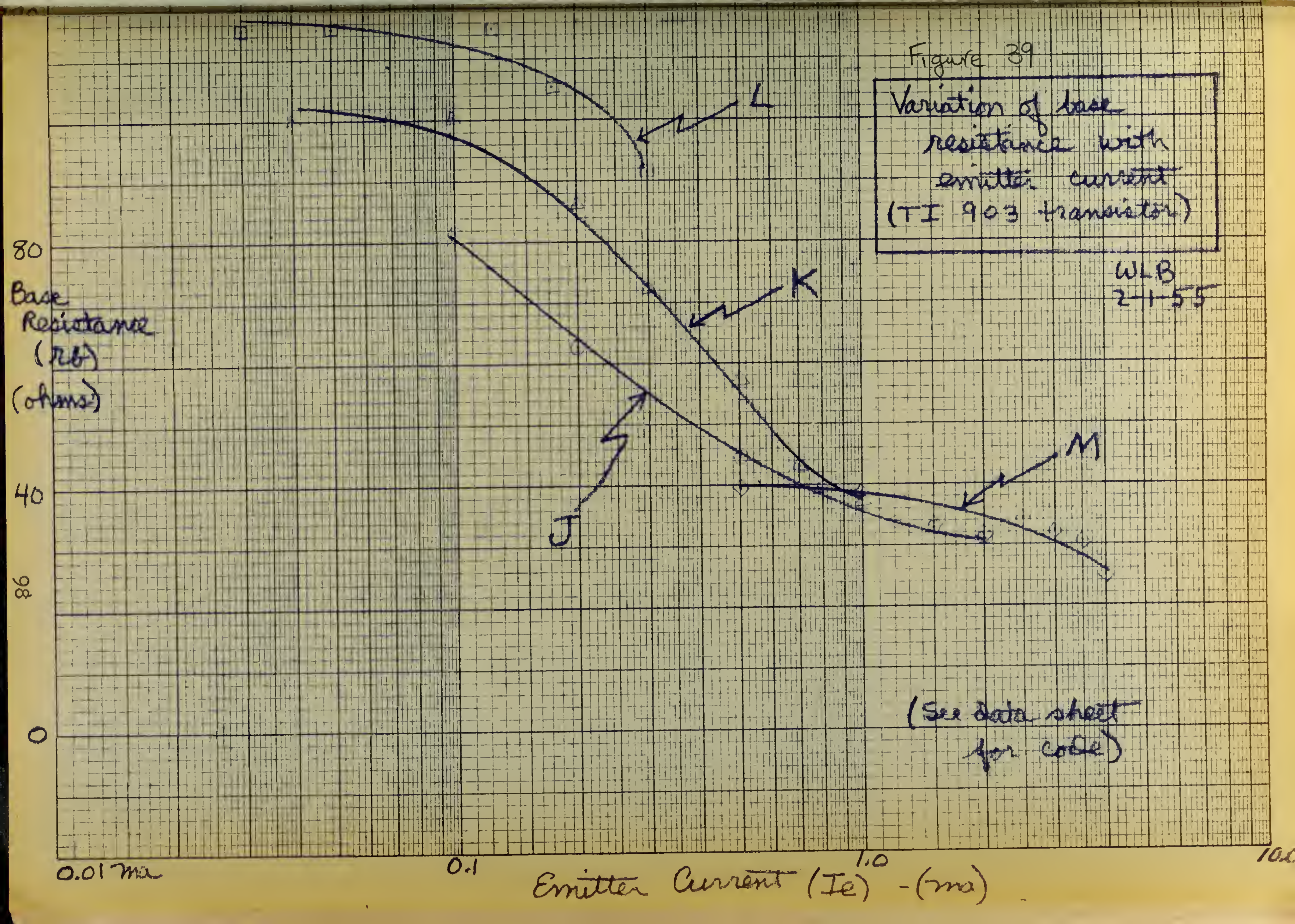
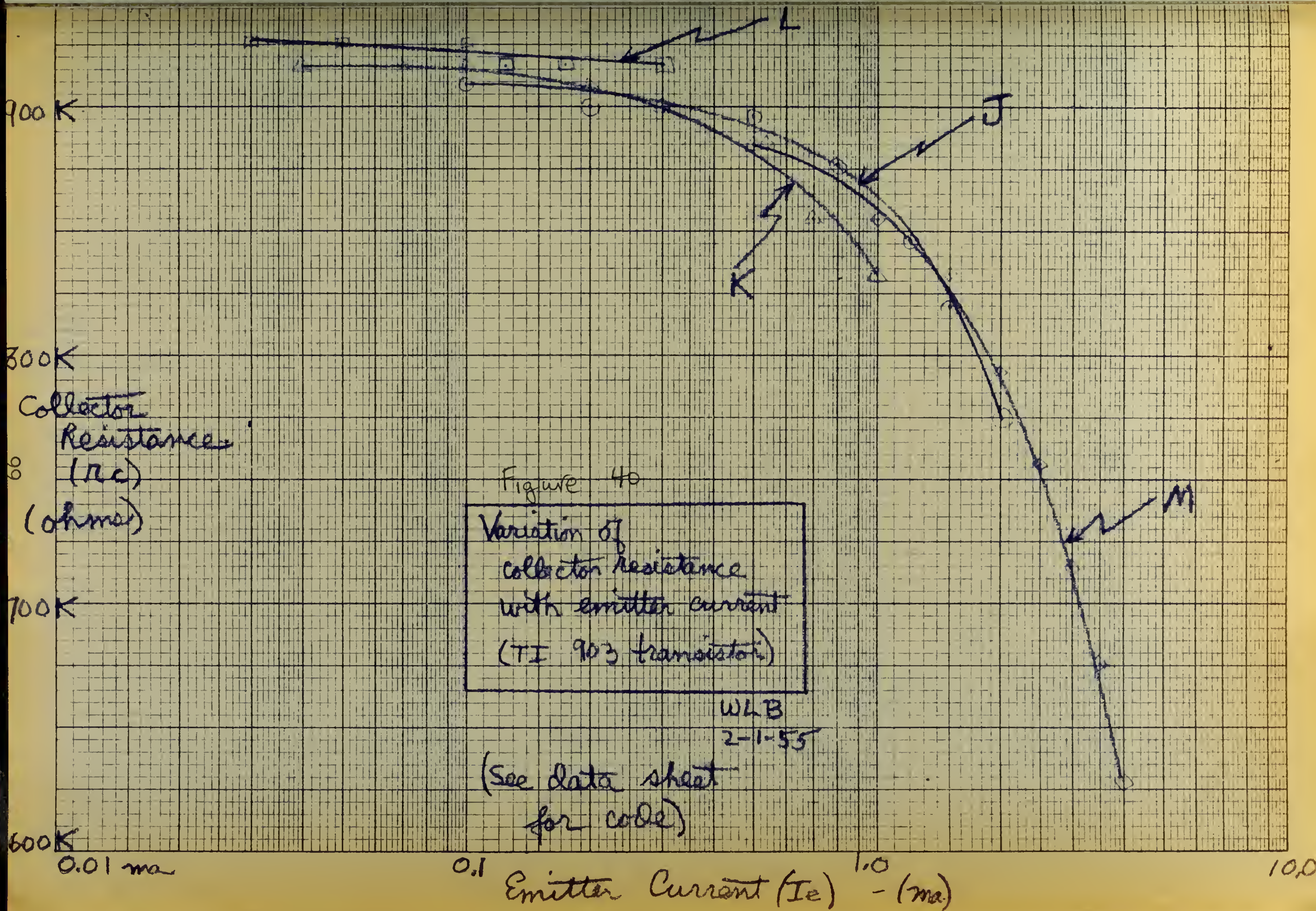
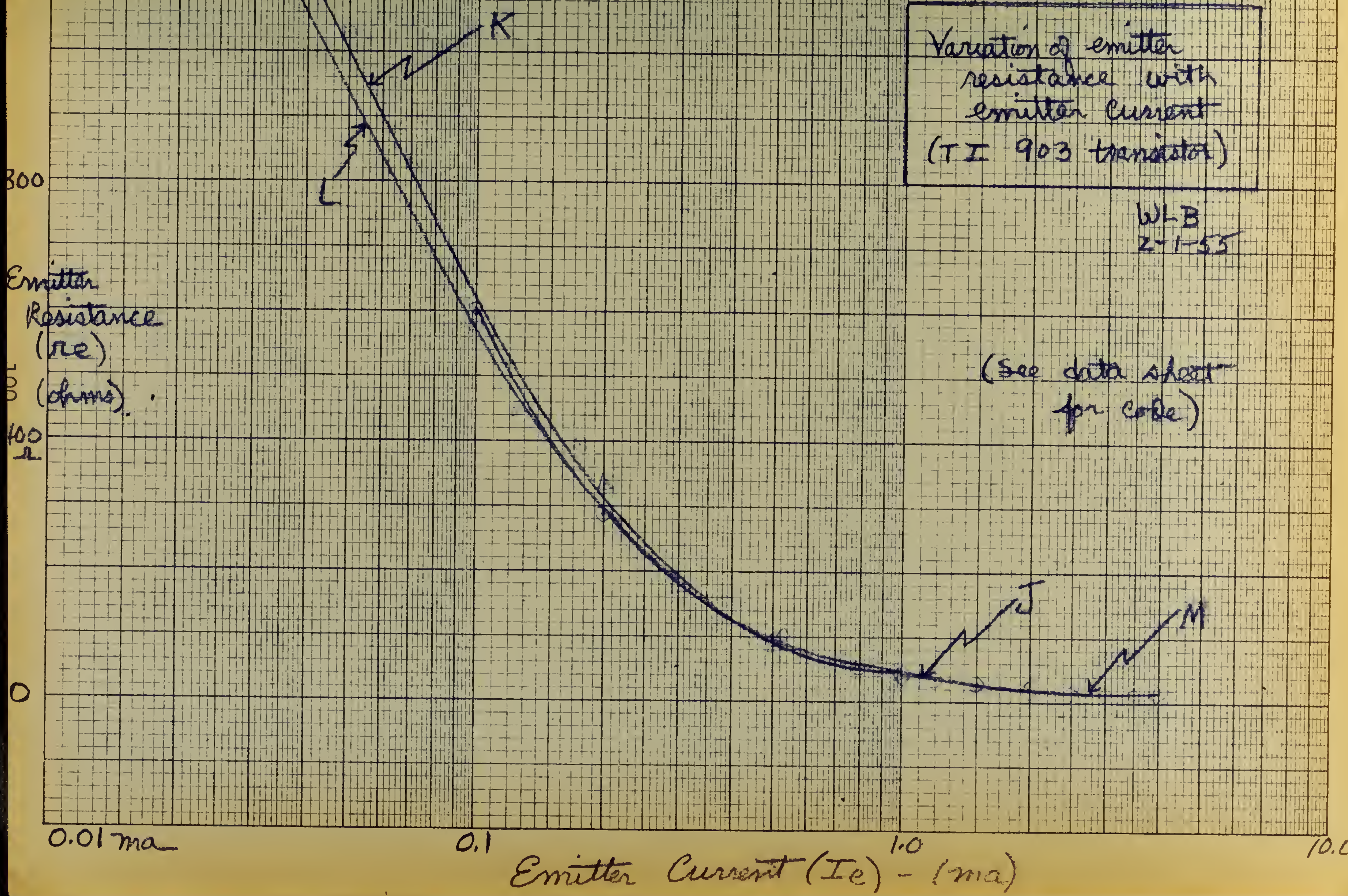


Figure 39
Variation of base resistance with emitter current (TI 903 transistor)
WLB
2-1-55
L
K
J
M
80
Base Resistance (rb) (ohms)
40
98
0
0.01 ma
0.1
1.0
10.0
Emitter Current (Ie) -(ma)
(See data sheet for code)



900 K
800 K
700 K
600 K
Collector Resistance (r_c) (ohms)
0.01 ma
0.1
1.0
10.0
Emitter Current (I_e) - (ma)
L
J
K
M
Figure 40
Variation of collector resistance with emitter current (TI 903 transistor)
WLB
2-1-55
(See data sheet for code)

Figure 41

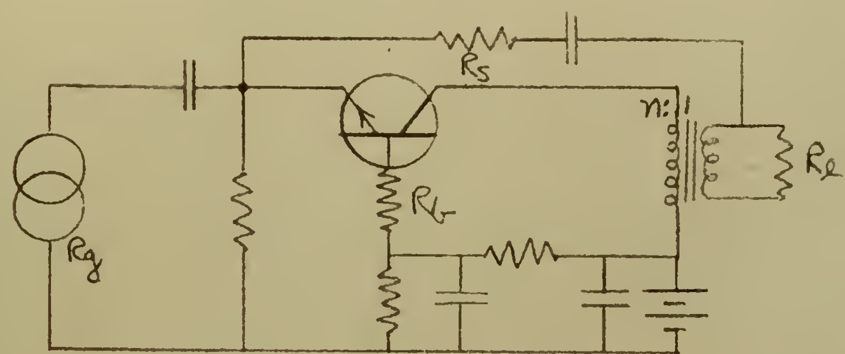


Variation of emitter resistance with emitter current (TI 903 transistor)
WLB
2-1-55
(See data sheet for code)
K
L
J
M
300
100
0
Emitter Resistance (re) (ohms)
0.01 ma
0.1
1.0
10.0
Emitter Current (Ie) - (ma)

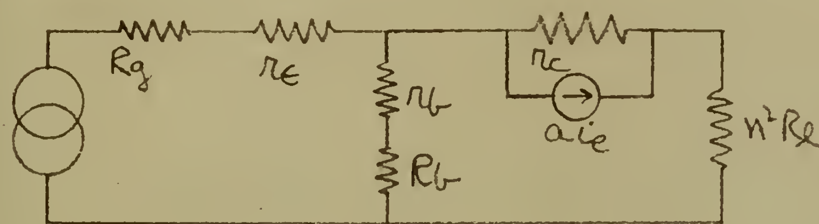
APPENDIX II

FEEDBACK IN TRANSISTOR AMPLIFIERS

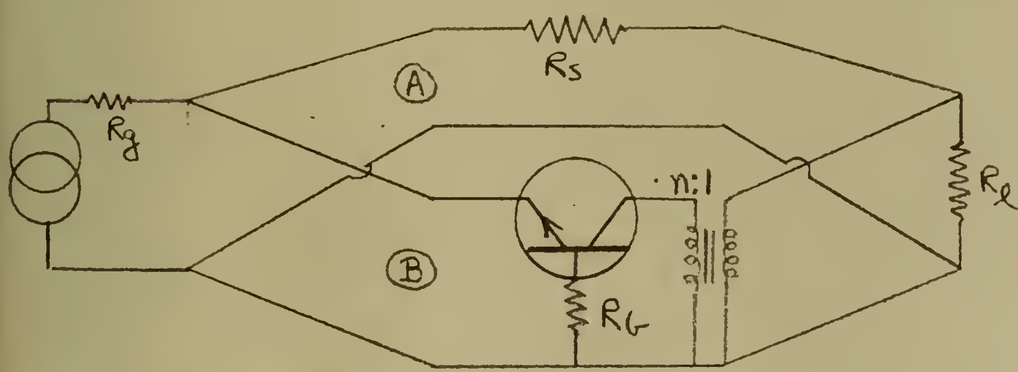
A. Common Base Configuration



Common Base Amplifier with Series and Shunt Feedback



T Network equivalent circuit for series feedback only



A-c equivalent circuit for series and shunt feedback

Figure 42

APPENDIX II

Assumptions:

$$a \approx \alpha \qquad r_c \gg n^2 R_e$$
$$r_c \gg r_e \qquad r_c \gg R_b$$
$$r_c \gg r_b \qquad r_c \gg R_g$$
$$R_b \gg r_b$$

1. Series Feedback

a. Input resistance

$$r_{i_{se}} = r_e + (r_b + R_b)\left[\frac{r_c(1-a) + n^2 R_e}{r_b + R_b + r_c + n^2 R_e}\right]$$

$$\approx r_e + R_b\left[\frac{r_c(1-a) + n^2 R_e}{r_c}\right] \qquad (\text{II-1})$$

$$\frac{r_{i_{se}}}{r_i} = \frac{r_e + (r_b + R_b)\left[\dfrac{r_c(1-a) + n^2 R_e}{r_b + R_b + r_c + n^2 R_e}\right]}{r_e + r_b\left[\dfrac{r_c(1-a) + n^2 R_e}{r_b + r_c + n^2 R_e}\right]}$$

$$\approx \frac{r_e + (r_b + R_b)\left[\dfrac{r_c(1-a) + n^2 R_e}{r_c}\right]}{r_e + r_b\left[\dfrac{r_c(1-a) + n^2 R_e}{r_c}\right]}$$

$$\approx 1 + R_b\left\{\frac{r_c(1-a) + n^2 R_e}{r_c\left[r_e + r_b(1-a)\right] + r_b n^2 R_e}\right\}$$

$$\frac{r_{ise}}{r_i} \approx 1 + R_b \left[\frac{1}{r_b + \dfrac{r_e r_c}{r_c(1-a) + n^2 R_\ell}} \right] \qquad (\text{II-2})$$

b. Current amplification

$$A_{ise} = \frac{a r_c + r_b + R_b}{r_b + R_b + r_c + n^2 R_\ell}$$

$$\approx \frac{a r_c}{r_c} \approx a \qquad (\text{II - 3})$$

$$\frac{A_{ise}}{A_i} \approx 1 \qquad (\text{II - 4})$$

c. Power gain

$$G_{se} = \frac{A_{i_{se}}^2 n^2 R_\ell}{r_{ise}}$$

$$\approx \frac{a^2 n^2 R_\ell}{r_e + R_b \left[\dfrac{r_c(1-a) + n^2 R_\ell}{r_c} \right]}$$

$$\approx \frac{a^2 r_c n^2 R_\ell}{\left[r_c(1-a) + n^2 R_\ell \right] \left[\dfrac{r_e r_c}{r_c(1-a) + n^2 R_\ell} + R_b \right]} \qquad (\text{II - 5})$$

$$\frac{G_{se}}{G} = \frac{A_{i_{se}}^2 r_i}{A_i^2 r_{ise}}$$

$$\frac{G_{se}}{G} \approx \frac{1}{1 + R_b\left[\dfrac{1}{r_b + \dfrac{r_e r_c}{r_c(1-a) + n^2 R_l}}\right]} \qquad (II-6)$$

2. Series and Shunt Feedback
   (* indicates sign of the term reversed for negative shunt feedback)

   a. Input resistance

$$[y]_A = \begin{bmatrix} \frac{1}{R_s} & -\frac{1}{R_s} \\ -\frac{1}{R_s} & \frac{1}{R_s} \end{bmatrix}$$

$$[y]_B = \frac{1}{\Delta}\begin{bmatrix} r_b + R_b + r_c & -n(r_b+R_b)^{*} \\ -n(r_b+R_b+ar_c)^{*} & n^2(r_e+R_b+r_b) \end{bmatrix}$$

$$\text{where} \quad \Delta = r_e(r_b+R_b) + r_c\left[r_e + (r_b+R_b)(1-a)\right]$$
$$\approx r_c R_b (1-a)$$

Now
$$r_i = \frac{y_{22} + y_l}{\Delta^y + y_{11} y_l}$$

$$= \frac{\left\{\dfrac{n^2(r_e+R_b+r_b)}{r_c R_b(1-a)} + \dfrac{1}{R_s} + \dfrac{1}{R_l}\right\}}{\left\{\begin{array}{l}\dfrac{n^2(r_e+R_b+r_b)(r_b+R_b+r_c) - n^2(r_b+R_b)(r_b+R_b+ar_c)}{r_c^2 R_b^2(1-a)^2} \\ + \dfrac{1}{R_s}\left[\dfrac{r_b+R_b+r_c-2nr_b^{*}-2nR_b^{*}-nar_c^{*} + n^2(r_e+R_b+r_b)}{r_c R_b(1-a)}\right] \\ + \dfrac{1}{R_l}\left[\dfrac{r_b+R_b+r_c}{r_c R_b(1-a)} + \dfrac{1}{R_s}\right]\end{array}\right\}}$$

$$r_i \approx \frac{\left\{\dfrac{n^2 R_s R_\ell + r_c(1-a)(R_s+R_\ell)}{r_c(1-a)\,R_s R_\ell}\right\}}{\left\{\dfrac{R_s R_\ell\, n^2 R_b r_c(1-a) + r_c R_b(1-a)R_\ell\left[r_c(1-n\overset{*}{a}) + n^2 R_b\right] + r_c R_b(1-a)\left[r_c R_s + r_c R_b(1-a)\right]}{r_c^2 R_b^2 (1-a)^2 R_s R_\ell}\right\}}$$

$$\approx \frac{\left\{\dfrac{\left[n^2 R_\ell + r_c(1-a)\right]R_s + r_c(1-a)R_\ell}{r_c(1-a)\,R_s R_\ell}\right\}}{\left\{\dfrac{r_c R_s + \left[n^2 R_\ell + r_c(1-a)\right]R_b + r_c(1-n\overset{*}{a})R_\ell}{r_c R_b(1-a)\,R_s R_\ell}\right\}}$$

$$\approx \frac{\left[n^2 R_\ell + r_c(1-a) + \dfrac{r_c(1-a)R_\ell}{R_s}\right]R_b}{\left\{r_c + \dfrac{\left[n^2 R_\ell + r_c(1-a)\right]}{R_s}R_b + \dfrac{r_c(1-na^*)}{R_s}R_\ell\right\}} \qquad (\text{II}-7)$$

b. Current amplification

$$A_i = \frac{-y_{21}\, y_\ell}{\Delta^y + y_{11} y_\ell}$$

The denominator of this function is identical to that of the input resistance. [illegible] proceed to evaluate the numerator of the fraction.

The numerator is equal to:

$$= -\left[-\frac{1}{R_s} - \frac{n(r_b + R_b + a r_c)^*}{r_c R_b (1-a)}\right]\frac{1}{R_l}$$

$$\approx \frac{R_b(1-a) + naR_s^*}{R_b R_s R_l (1-a)}$$

$$\therefore A_i \approx \frac{r_c\left[R_b\frac{(1-a)}{R_s} + na^*\right]}{\left\{r_c + \frac{n^2 R_l + r_c(1-a)}{R_s}R_b + \frac{r_c(1-na^*)}{R_s}R_l\right\}} \qquad \text{(II-8)}$$

c. Power gain

$$G = \frac{A_i^2 R_l}{r_i}$$

$$\approx \frac{r_c^2\left[R_b\frac{(1-a)}{R_s} + na^*\right]^2 R_l}{\left\{\left[n^2 R_l + r_c(1-a) + \frac{r_c(1-a)R_l}{R_s}\right] \times R_b\left[r_c + \frac{n^2 R_l + r_c(1-a)}{R_s}R_b + \frac{r_c(1-na^*)}{R_s}R_l\right]\right\}} \qquad \text{(II-9)}$$

3. Shunt feedback:
(* indicates sign of the term reversed for positive shunt feedback)

The formulas for the desired quantities must be reevaluated from the original matrices with Rb equal to zero.

Note that now:

$$\Delta \approx r_c\left[r_e + r_b(1-a)\right]$$

APPENDIX II

a. Input resistance

$$r_i \approx \frac{\left\{ \dfrac{n^2(r_e+r_b)}{r_c\left[r_e+r_b(1-a)\right]} + \dfrac{1}{R_s} + \dfrac{1}{R_l} \right\}}{\left\{ \dfrac{n^2(r_e+r_b)r_c - n^2 r_b a r_c}{r_c^2\left[r_e+r_b(1-a)\right]^2} + \dfrac{1}{R_s}\left[\dfrac{r_c(1-na^*)+n^2(r_e+r_b)}{r_c\left[r_e+r_b(1-a)\right]}\right] + \dfrac{1}{R_l}\left[\dfrac{1}{r_e+r_b(1-a)} + \dfrac{1}{R_s}\right] \right\}}$$

$$\approx \frac{\left\{ \dfrac{n^2 R_l R_s(r_e+r_b) + r_c\left[r_e+r_b(1-a)\right](R_s+R_l)}{r_c R_s R_l\left[r_e+r_b(1-a)\right]} \right\}}{\left\{ \dfrac{n^2 R_l R_s r_c\left[r_e+r_b(1-a)\right] + R_l r_c\left[r_e+r_b(1-a)\right] r_c(1-na^*) + r_c\left[r_e+r_b(1-a)\right]\left[r_c R_s + r_e + r_b(1-a)\right]}{r_c^2\left[r_e+r_b(1-a)\right]^2 R_s R_l} \right\}}$$

$$\approx \frac{\left\{ \dfrac{\left\{r_c r_e + r_b\left[n^2 R_l + r_c(1-a)\right]\right\} R_s + r_c R_l\left[r_e+r_b(1-a)\right]}{r_c R_s R_l\left[r_e+r_b(1-a)\right]} \right\}}{\left\{ \dfrac{r_c R_s + r_c(1-na^*)R_l + r_e + r_b(1-a)}{r_c\left[r_e+r_b(1-a)\right] R_s R_l} \right\}}$$

$$r_i \approx \frac{\left\{ r_c\left[r_e + r_b(1-a)\right] + n^2 R_l r_b + \dfrac{r_c\left[r_e + r_b(1-a)\right]}{R_s} R_l \right\}}{r_c\left[1 + \dfrac{(1-na^*)}{R_s} R_l\right]} \qquad (\text{II}-10)$$

b. Current amplification

The reciprocal of this expression is:

$$= -\left[-\frac{1}{R_s} - \frac{n(r_b + a r_c)^*}{r_c\left[r_e + r_b(1-a)\right]}\right]\frac{1}{R_l}$$

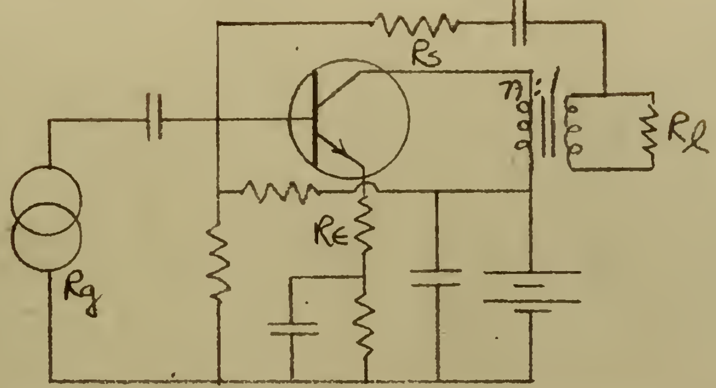
$$\approx r_c\left[\frac{r_e + r_b(1-a) + n a R_s^*}{R_s R_l\, r_c\left[r_e + r_b(1-a)\right]}\right]$$

$$\therefore A_i \approx \frac{\left\{\dfrac{r_e + r_b(1-a)}{R_s} + na^*\right\}}{\left\{1 + \dfrac{1-na^*}{R_s} R_l\right\}} \qquad (\text{II}-11)$$

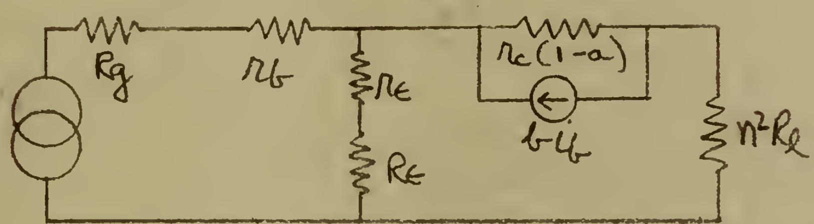
c. Power gain

$$G \approx \frac{\left[na^* + \dfrac{r_e + r_b(1-a)}{R_s}\right]^2 R_l r_c}{\left\{\left[1 + \dfrac{1-na^*}{R_s} R_l\right] \times \left[r_c\left[r_e + r_b(1-a)\right] + n^2 R_l r_b + \dfrac{r_c\left[r_e + r_b(1-a)\right]}{R_s} R_l\right]\right\}} \qquad (\text{II}-12)$$

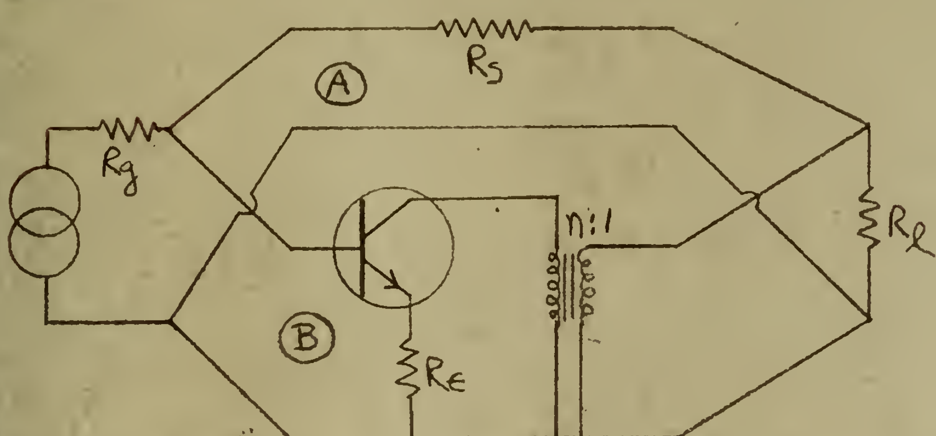
B. Common Emitter Configuration



Common Emitter Amplifier with Series and Shunt Feedback



T Network equivalent circuit for series feedback only



A-c equivalent circuit for series and shunt feedback

Figure 43

Assumptions: $a \approx \alpha$ $\qquad r_c \gg n^2 R_l$

$r_c \gg r_e \qquad r_c \gg R_E$

$r_c \gg r_b \qquad r_c \gg R_g$

$r_c(1-a) \gg r_e$

1. Series Feedback

a. Input resistance

$$r_{i_{se}} = r_b + (r_e + R_E)\frac{r_c + n^2 R_l}{r_c(1-a) + r_e + R_E + n^2 R_l}$$

$$\approx r_b + \frac{r_c}{1 + \dfrac{r_c(1-a) + n^2 R_l}{r_e + R_E}} \qquad (\text{II-13})$$

$$\frac{r_{i_{se}}}{r_i} = \frac{r_b + (r_e + R_E)\dfrac{r_c + n^2 R_l}{r_c(1-a) + r_e + R_E + n^2 R_l}}{r_b + r_e\dfrac{r_c + n^2 R_l}{r_c(1-a) + r_e + n^2 R_l}}$$

$$\approx \frac{1 + R_E\dfrac{r_c}{r_c[r_e + r_b(1-a)] + r_b n^2 R_l}}{1 + R_E\dfrac{1}{r_c(1-a) + n^2 R_l}} \qquad (\text{II-14})$$

b. Current amplification

$$A_{i_{se}} = \frac{a r_c - r_e - R_E}{r_c(1-a) + r_e + R_E + n^2 R_l}$$

$$\approx \frac{a r_c}{r_c(1-a) + R_E + n^2 R_l} \qquad (\text{II-15})$$

$$\frac{A_{ise}}{A_i} = \frac{(ar_c - r_\epsilon - R_\epsilon)(r_c[1-a] + r_\epsilon + n^2 R_\ell)}{(r_c[1-a] + r_\epsilon + R_\epsilon + n^2 R_\ell)(ar_c - r_\epsilon)}$$

$$\approx \frac{1 - \dfrac{R_\epsilon}{a r_c}}{1 + \dfrac{R_\epsilon}{r_c(1-a) + n^2 R_\ell}}$$

$$\approx \frac{1}{1 + R_\epsilon \dfrac{1}{r_c(1-a) + n^2 R_\ell}} \qquad (\text{II-16})$$

c. Power gain

$$G_{se} = \frac{A_{i\,se}^2 \, n^2 R_\ell}{r_{i\,se}}$$

$$\approx \frac{a^2 r_c^2 n^2 R_\ell \left[r_c(1-a) + R_\epsilon + n^2 R_\ell\right]}{\left[r_c(1-a) + R_\epsilon + n^2 R_\ell\right]^2 \left[r_b\{r_c(1-a) + R_\epsilon + n^2 R_\ell\} + r_c(R_\epsilon + r_\epsilon)\right]}$$

$$\approx \frac{a^2 r_c^2 n^2 R_\ell}{\left[r_c(1-a) + n^2 R_\ell + R_\epsilon\right]\left[r_c\{R_\epsilon + r_\epsilon + r_b(1-a)\} + r_b n^2 R_\ell\right]} \qquad (\text{II-17})$$

$$\frac{G_{se}}{G} = \frac{A_{ise}^2 \, r_i}{A_i^2 \, r_{se}}$$

$$\frac{G_{se}}{G} \approx \left(\frac{1}{1 + \dfrac{R_\epsilon}{r_c(1-a)+n^2R_l}} \right)^2 \left(\frac{\cancel{1 + \dfrac{R_\epsilon}{r_c(1-a)+n^2R_l}}}{1 + \dfrac{r_c R_\epsilon}{r_b[r_c(1-a)+n^2R_l]+r_c r_\epsilon}} \right)$$

$$\approx \frac{1}{\left(1 + \dfrac{R_\epsilon}{r_c(1-a)+n^2R_l}\right)\left(1 + R_\epsilon \dfrac{r_c}{r_b[r_c(1-a)+n^2R_l] + r_c r_\epsilon}\right)} \qquad (\text{II-18})$$

2. Series and Shunt Feedback
(* indicates sign of the term reversed for negative shunt feedback)

a. Input resistance

$$[y]_A = \begin{bmatrix} \dfrac{1}{R_s} & -\dfrac{1}{R_s} \\ -\dfrac{1}{R_s} & \dfrac{1}{R_s} \end{bmatrix}$$

$$[y]_B = \begin{bmatrix} \dfrac{r_\epsilon + R_\epsilon + r_c(1-a)}{\Delta} & \dfrac{+n(r_\epsilon + R_\epsilon)^*}{\Delta} \\ \dfrac{+n(r_\epsilon + R_\epsilon - a r_c)^*}{\Delta} & \dfrac{n^2(r_\epsilon + R_\epsilon + r_b)}{\Delta} \end{bmatrix}$$

where $\Delta = (r_\epsilon + R_\epsilon) r_b + r_c[r_\epsilon + R_\epsilon + r_b(1-a)]$

$$\approx (r_\epsilon + R_\epsilon) r_c + r_c r_b (1-a)$$

$$\approx r_c[r_\epsilon + R_\epsilon + r_b(1-a)]$$

Let $\quad F \triangleq r_\epsilon + R_\epsilon + r_b(1-a)$

So that $\quad \Delta \approx r_c F$

Now $\quad r_i = \dfrac{y_{22} + y_l}{\Delta y + y_{11}\, y_l}$

$$= \frac{\left\{ \dfrac{n^2(r_\epsilon + R_\epsilon + r_b)}{r_c F} + \dfrac{1}{R_s} + \dfrac{1}{R_l} \right\}}{\left\{ \begin{aligned} &\frac{n^2(r_\epsilon + R_\epsilon + r_b)(r_\epsilon + R_\epsilon + r_c[1-a]) - n^2(r_\epsilon + R_\epsilon)(r_\epsilon + R_\epsilon - a r_c)}{r_c^2 F^2} \\ &+ \frac{1}{R_s}\left\{ \frac{r_\epsilon + R_\epsilon + r_c(1-a) + n(2r_\epsilon + 2R_\epsilon - a r_c)^{*} + n^2(r_\epsilon + r_b)}{r_c F} \right\} \\ &+ \frac{1}{R_l}\left\{ \frac{r_\epsilon + R_\epsilon + r_c(1-a)}{r_c F} + \frac{1}{R_s} \right\} \end{aligned} \right\}}$$

$$\approx \frac{\left\{ \dfrac{n^2(r_\epsilon + R_\epsilon + r_b) R_s R_l + r_c F (R_s + R_l)}{r_c F R_s R_l} \right\}}{\left\{ \begin{aligned} \frac{1}{r_c^2 F^2 R_s R_l} \Big\{ & R_s R_l \left[n^2 \{R_\epsilon + r_c(1-a)\}\{r_\epsilon + R_\epsilon + r_b\} + n^2(r_\epsilon + R_\epsilon) a r_c \right] \\ & + r_c F R_l \left(-n a r_c^{*}\right) \\ & + r_c F \left[\{r_c(1-a) + R_\epsilon\} R_s + r_c F \right] \Big\} \end{aligned} \right\}}$$

$$
r_i \approx \frac{\left\{ \dfrac{\left\{ \left[n^2 R_l + r_c(1-a) \right] r_b + r_c(r_\epsilon + R_\epsilon) \right\} R_s + R_l r_c F}{r_c F R_s R_l} \right\}}{\left\{ \dfrac{R_s R_l \left[n^2 (r_c F + R_\epsilon r_b) \right] + r_c^2 F \left[(1-a) R_s + F - na R_l^* \right] + r_c F R_\epsilon R_s}{r_c^2 F^2 R_s R_l} \right\}}
$$

$$
\approx \frac{\left\{ \dfrac{\left(n^2 R_l r_b + r_c F \right) R_s + r_c F R_l}{r_c F R_s R_l} \right\}}{\left\{ \dfrac{\left\{ r_c F \left[n^2 R_l + r_c(1-a) + R_\epsilon \right] + R_\epsilon r_b n^2 R_l \right\} R_s + r_c^2 F \left[F - na R_l^* \right]}{r_c^2 F^2 R_s R_l} \right\}}
$$

$$
\approx \frac{\left\{ n^2 R_l r_b + r_c \left[r_\epsilon + R_\epsilon + r_b(1-a) \right] + \dfrac{r_c R_l \left[r_\epsilon + R_\epsilon + r_b(1-a) \right]}{R_s} \right\}}{\left\{ n^2 R_l + r_c(1-a) + R_\epsilon + \dfrac{R_\epsilon r_b n^2 R_l}{r_c(r_\epsilon + R_\epsilon + r_b[1-a])} + \dfrac{r_c \left(R_\epsilon - na R_l^* \right)}{R_s} \right\}} \qquad (\text{II-19})
$$

b. Current Amplification

$$A_i = \frac{-y_{21}\, y_l}{\Delta^y + y_{11}\, y_l}$$

The denominator of this fraction is identical to that of the input resistance. We therefore proceed to evaluate the numerator of the fraction, this being equal to:

$$= -\left[-\frac{1}{R_s} + \frac{n(r_\epsilon + R_\epsilon - a r_c)^*}{r_c F}\right]\frac{1}{R_l}$$

$$\approx \frac{n R_s a r_c^* + r_c F}{r_c F R_s R_l} \approx \frac{F + a n R_s^*}{F R_s R_l}$$

$$\approx \frac{R_\epsilon + a n R_s^*}{F R_s R_l}$$

$$\therefore A_i \approx \frac{r_c\left(\frac{R_\epsilon}{R_s} + a n^*\right)}{\left\{ n^2 R_l + r_c(1-a) + R_\epsilon + \frac{R_\epsilon r_b n^2 R_l}{r_c(r_\epsilon + R_\epsilon + r_b[1-a])} + \frac{r_c(R_\epsilon - n a R_l^*)}{R_s} \right\}} \qquad (II\text{-}20)$$

c. Power gain

$$G = \frac{A_i^{\,2} R_l}{r_i}$$

$$G \approx \frac{r_c^2\left(\frac{R_e}{R_s} + an^*\right)^2 R_l}{\left\{\begin{array}{l}\left[n^2 R_l r_b + r_c\left[r_e + R_e + r_b(1-a)\right] + \frac{r_c R_l\left[r_e + R_e + r_b(1-a)\right]}{R_s}\right] \\ \times \left[n^2 R_l + r_c(1-a) + R_e + \frac{R_e r_b n^2 R_l}{r_c(r_e + R_e + r_b[1-a])} + \frac{r_c(R_e - naR_l^*)}{R_s}\right]\end{array}\right\}}$$

(II-21)

3. Shunt Feedback
   (* indicates sign of the term reversed for negative feedback)

The desired formulas may be obtained simply by letting Re approach zero in formulas (II-19, II-20, and II-21).

Therefore:

a. Input resistance

$$r_i \approx \frac{\left\{n^2 R_l r_b + r_c\left[r_e + r_b(1-a)\right] + \frac{r_c R_l\left[r_e + r_b(1-a)\right]}{R_s}\right\}}{n^2 R_l + r_c(1-a) - \frac{naR_l r_c^*}{R_s}} \quad \text{(II-22)}$$

b. Current Amplification

$$A_i \approx \frac{a n r_c^*}{n^2 R_l + r_c(1-a) - \dfrac{a n R_l r_c^*}{R_s}} \qquad \text{(II-23)}$$

c. Power gain

$$G \approx \frac{a^2 n^2 R_l r_c^2}{\left\{ \left(n^2 R_l r_b + r_c[r_e + r_b(1-a)] + \dfrac{r_c R_l [r_e + r_b(1-a)]}{R_s} \right) \times \left(n^2 R_l + r_c(1-a) - \dfrac{n a R_l r_c^*}{R_s} \right) \right\}}$$

APPENDIX III

DERIVATION SHOWING THE EFFECT OF POSITIVE SHUNT FEEDBACK ON THE RATE OF CHANGE OF INSERTION GAIN OF A COMMON EMITTER AMPLIFIER WITH SERIES FEEDBACK RESISTANCE

Circuit: see diagram under Section B, Appendix II.

We desire that $\dfrac{\partial G(R_S, R_E)}{\partial R_E}$ be a maximum. This derivative is itself a function of Re and Rs, and we desrie to observe the effect of Rs on it. Thus we find: $\dfrac{\partial}{\partial R_S}\left(\dfrac{\partial G}{\partial R_E}\right)$.

From formula (II-21) in the region where:

$$R_E \ll a n R_S$$

$$R_\ell \ll R_S$$

$$R_S \ll r_c,$$

$$G \approx \frac{a^2 r_c^2 n^2 R_\ell}{\left\{\left[n^2 R_\ell + r_c(1-a)\right] r_b + r_c r_e + r_c R_E\right\} \times \left\{n^2 R_\ell + r_c(1-a) + \dfrac{r_c(R_E - a n R_\ell)}{R_S}\right\}}$$

Let $K \triangleq n^2 R_\ell + r_c(1-a)$

a positive number for a junction transistor.

Then

$$G \approx \frac{a^2 r_c^2 n^2 R_\ell R_s}{(K r_b + r_c r_\epsilon + r_c R_\epsilon)(K R_s + r_c[R_\epsilon - n a R_\ell])}$$

$$\frac{\partial G}{\partial R_\epsilon} = \frac{a^2 r_c^2 n^2 R_\ell R_s \left[-r_c \left(K R_s + r_c \{R_\epsilon - n a R_\ell\}\right) - r_c \left(K r_b + r_c r_\epsilon + r_c R_\epsilon\right)\right]}{(K r_b + r_c r_\epsilon + r_c R_\epsilon)^2 (K R_s + r_c [R_\epsilon - n a R_\ell])^2}$$

Let $L \triangleq K r_b + r_c r_\epsilon + r_c R_\epsilon$

$M \triangleq a^2 r_c^3 n^2 R_\ell R_s$

both positive numbers

Then

$$\frac{\partial}{\partial R_s}\left(\frac{\partial G}{\partial R_\epsilon}\right) = -\frac{M\left[K R_s + r_c (R_\epsilon - n a R_\ell) + L\right]}{L^2 \left(K R_s + r_c [R_\epsilon - n a R_\ell]\right)^2}$$

$$= -\frac{M\left\{K\left(K R_s + r_c [R_\epsilon - n a R_\ell]\right) - 2K\right\}}{L^2 \left(K R_s + r_c [R_\epsilon - n a R_\ell]\right)^3}$$

This derivative will always be negative if:

$$K R_s + r_c R_\epsilon > 2 + r_c a n R_\ell$$

$$[n^2 R_\ell + r_c (1-a)] R_s + r_c R_\epsilon > r_c a n R_\ell$$

$$r_c[(1-a) R_s + R_\epsilon - a n R_\ell] + n^2 R_\ell R_s > 0 \qquad (\text{III-1})$$

Therefore, provided that inequality (III-1) holds, $\frac{\partial}{\partial R_S}\left(\frac{\partial G}{\partial R_E}\right)$ is always negative, that is, an increase in positive feedback by decreasing Rs increases the slope of the gain versus series feedback resistance curve.

From formula (II-19) in this same region,

$$r_i \approx \frac{\left[n^2 R_\ell + r_c(1-a)\right] r_b + (r_e + R_E)\, r_c}{n^2 R_\ell + r_c(1-a) + \dfrac{r_c(R_E - a n R_\ell)}{R_S}}$$

$$\approx \frac{(K r_b + r_e r_c + R_E r_c)\, R_S}{r_c\left[(1-a)R_S + R_E - a n R_\ell\right] + n^2 R_\ell R_S}$$

Note that when inequality (III-1) does not hold, the input resistance is negative.